Modelling and upper bound analysis of involute spur gear precision forging

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ABSTRACT: Geometric modelling of involute spur gears, precision forging process simulation and upper bound analysis are the subjects of this paper. Computer programs for the geometric modelling have been written with Visual Basic in SolidWorks macro. Module, teeth number, pressure angle, gear height and bore radius of the spur gear were input to a parametric computer program and then spur gear solid model is constructed. Using spur gear model, billet dimensions, the die cavity, the punch and the ejector are modelled automatically. Forming load is calculated using the upper bounds. For upper bound analyses, half pitch of a tooth with involute curve has been divided into six deformation zones. A velocity field in which bulging of the tooth form is considered has been proposed. The obtained forging load was compared with experimental results carried out in literature and simulated with SuperForge commercial code.

Keywords: Precision forging, spur gear, geometric modelling, upper bound, bulge

INTRODUCTION

Spur gears are used to transmit rotary motion and torque between parallel shafts; the teeth are straight and parallel to the axis of rotation. Involute spur gears are widely applied in reducers, transmissions and many other industrial applications. Close die forging and cutting are the traditional processes for producing spur gears. Precision forging of spur gears has been the subject of considerable effort in the last few decades in the forging industry. The manufacturing procedure of precision forging includes advantages of saving cutting cost, less raw material, saving time, lower power dissipation, and improved strength of teeth and fatigue limit of die (Abdul and Dean, 1998). For the forging of spur gears, predicting the power and load requirement is an important feature of the forging process.

The precision forging of spur gears has been explored by Abdul and Dean (1998) and Chitkara and Bhutta (1995) using the upper bound method. Through the analysis, the tooth profile has been assumed as a straight line parallel to the centerline of the tooth and neglected bulging. Choi et al. (1996, 2000) and Cho et al. (1997) also used an involute tooth profile instead of the tapered profile to obtain a more realistic solution in their upper bound analysis but they neglected bulging.

Assuming straight line as the tooth profile, Chitkara and Bhutta (1999) and El-Domaity and Shabara (1998) have analyzed gear forging by slab method and compared the results with that obtained by the upper bound method. Hsu (2002) and Chitkara and Yohngjo (1996) by using the upper bound method has studied forging process of spur gears. They predicted the load requirement and the metal flow during the closed die forging of spline and spur gear form which is of a straight tapered tooth profile by the upper bound method. Hsu has considered bulging but assuming tooth profile like a straight line. Sadeghi (2003) has been modelled the process of precision forging spur and helical gears by means of upper bound analysis. He assumed the tooth profile to be trapezoidal and bulging of the tooth form included in his analysis. Chengliang et al. (2007) have used finite element model to investigate the complete filling of the die and gear teeth formation during the forging process. Kang et al. (2007) have studied elastic die deformations during the forging process and its effect on gear teeth accuracy. Plancak et al. (1992, 2003), Can et al. (2005) and Altinbalik and Can (2005) carried out analytical and experimental studies on spur gear forms and splines. Plancak et al. (1992) predicted the load requirement and the punch pressure for different segment angles by using the upper bound method.
with a simple velocity field. Later, Plancak et al. (2003) extended their analysis to three different tooth profiles namely radial, parallel and tapered. Altinbalik and Can (2005) predicted the load requirement for parallel toothed splines by using the an admissible velocity field and compared with experimental results.

In this paper, a parametric mathematical model is proposed by the upper bound method to investigate forging of billet material within toothed dies cavity by using solid billets with flat punch, and hollow billets with a punch with a mandrel. In the analysis, bulging is included and the involute curve has been used to represent the sides of gear teeth. Furthermore, comparisons between present theoretical results and experiments of other researchers’ work are carried out to illustrate the validity of this proposed model.

**Geometric modelling**

**Spur gear geometry**

Top view of a spur gear is shown in Figure 1 (a). A half tooth profile is shown in Figure 1 (b). This profile includes four curves: curve AB, curve BC, curve CD and curve DE. The curve AB is an arc of the root circle. It can be modelled using radius $r_r$ and angle $\theta_1$ as shown in Figure 1(b). These two parameters can be calculated using following equations (Mitra, 2000):

$$r_r = \frac{N-2}{2} M$$

$$\theta_1 = \frac{a}{2} + \tan^{-1}\left(\frac{\sqrt{r_f^2-r_b^2}}{r_b}\right) - \frac{\sqrt{r_f^2-r_b^2}}{r_b} - \cos^{-1}\left(\frac{r_f^2+2n_f+\frac{r_f^2}{2}}{2r_b(r_f+r_f)}\right)$$

Where $M$ is the module, $N$ is the number of teeth, $r_r$ is the radius of root circle, $\varphi$ is the pressure angle, $r_b$ is the radius of the base circle, $r_f$ is the fillet radius and $r_p$ is the radius of the pitch circle (shown in Figure 1(a)).

The curve of BC is fillet of the tooth and is tangent to curve AB and curve CD. The curve of DE is an arc of the addendum circle. Radius $r_f$ is the radius of the addendum circle.

The curve of the CD is involute form. An involute generated by unwinding a thread wrapped counter clockwise around the base circle would form the outer portion of the left sides of the teeth. At every point, the involute is perpendicular to the taut thread.

![Figure 1. A gear and the tooth profile geometry.](image)

All parameters shown in Figure 1 can be calculated using the following equations:

$$\alpha = \frac{\pi}{N}$$

$$\theta_f = \cos^{-1}\left(\frac{r_f^2+2n_f+r_f^2}{r_p^2(r_f+r_f)}\right)$$

$$\theta_s = \frac{a}{2} + \tan^{-1}\left(\frac{\sqrt{r_f^2-r_b^2}}{r_b}\right) - \frac{\sqrt{r_f^2-r_b^2}}{r_b}$$

**Gear, billet and die cavity modelling**

After modelling a half tooth profile, it is mirrored about centreline of the tooth (OE in Figure 1(b) ) and with a polar array of a tooth about point O, 2-D model of the spur gear will design, with extruding of this model equal to height of gear, gear solid model will designed.

The commonly used billet for the forging of a spur gear is a hollow cylindrical billet. The outer diameter of the billet is equal to that of the root circle. To achieve the complete filling of the form the billet must have the
exact volume needed for the gear. Too much material will cause an overflowing of the die and therefore an overload which may destroy the die.

Four working elements are required to forge hollow shapes; a punch, an ejector, a container and a mandrel. Die cavity is the female of the spur gear. The punch forms the top surface of the cavity and is attached to the moving ram of the forging machine.

The modelling characteristics of the gear die cavity and the die assembly have been expressed in parametric form and programmed.

**Upper bound analysis**

**Kinematically admissible velocity field**

The cylindrical coordinate system \((r, \theta, z)\) is used in this analysis (Figure 2(a)). The kinematically admissible velocity field should satisfy the incompressibility condition and boundary condition expressed as

\[
\dot{e}_{rr} + \dot{e}_{r\theta} + \dot{e}_{zz} = 0 
\]

And

\[
\dot{e}_{rr} = \frac{\partial U_r}{\partial r}, \quad \dot{e}_{r\theta} = \frac{1}{r} \left( \frac{\partial U_r}{\partial r} + U_r \right), \quad \dot{e}_{zz} = \frac{\partial U_z}{\partial z} 
\]

Where \(U_r\) is the radial velocity, \(U_\theta\) is the circumferential velocity and \(U_z\) is the axial velocity. The boundary condition is that on the surface of the workpiece the material should not flow across the die surface.

To analyze the forging of spur gears with \(N\) teeth, the gear was divided into \(2N\) deformation units. Figure 2 (b) shows a generic deforming unit bounded by two planes of symmetry with adjacent units. No metals can cross or shear along a plane of symmetry. Each of the deforming units, shown in Figure 2 (b), is then further subdivided, basically into six zones of deformation labelled I-VI, where the plastic flow is assumed to take place.

If the punch moves down with \(u/2\) and at any time the workpiece is assumed to have thickness \(a\) measured along \(z\) axis, then the velocity fields for regions I-IV are the same as choi and cho (1965) and for other regions are as follows.

Region V \((\theta_v + \pi \varphi_R \leq \theta \leq \alpha, r_v \leq r \leq R)\). The deformation in this region is not uniform and the free surface of the material bulges (Figure 2 (b)) due to the restraints at the contact surfaces. To predict the bulged surface profile of the tooth, the axial velocity component \(U_z\), which incorporate bulging, is assumed to take the form (Kobayashi et al., 1996):

\[
U_z = -K \left( 1 - B \frac{z^2}{a^2} \right) z
\]

Where \(B\) is a parameter between 0 and 1 and representing the severity of the bulge in the tooth region and \(K\) is a constant which can be determined from the velocity boundary condition that for \(z = a/2\), \(U_z = -u/2\)

Therefore:

\[
K = \frac{12u}{a(12-Ba^2)}
\]

To preserve continuity, the axial velocity component across the surface of velocity discontinuity IV-V and IV-VI, should remain unchanged. Therefore, it is assumed that no bulging takes place at this surface, i.e. \(B = 0\).

There is no resultant normal velocity on the involute curve, therefore:

\[
U_n = U_\tan \varphi_R
\]

Where
The shear power dissipated at the surface of velocity discontinuity is given by

\[ \text{power} = \frac{K}{2} (1 - B z^2) + \frac{C_s}{r} \tan \varphi \]

(16)

Where \( C_s \) is determined from the boundary conditions that for \( r = r_b, U_r = U_r^{L} \).

The expression becomes

\[ \text{power} = \frac{K}{2} (1 - B z^2) + \frac{C_s}{r} \tan \varphi \]

(17)

Region \( VI(\theta_s + \theta_{s+1}) \leq \theta \leq \theta_r, r_b \leq r \leq R \).

\[ U_r = \left[ \frac{K_v^2}{2B_r} \right] (1 - B z^2) \]

(18)

Where \( C_b \) is determined from the boundary conditions that for \( r = r_b, U_r = U_r^{L} \).

\[ U_r = \frac{K^2}{2B_r} \left( 1 - B z^2 \right) \]

(19)

\[ U_r = -K^2 (1 - B z^2) \]

(20)

The friction power loss at material/punch or ejector is given by

\[ \dot{W}_f = \frac{m_a}{2 \eta} \int_0^L \left| \Delta V \right| dS \]

(28)

Where \( \Delta V \) is the velocity discontinuity and \( dS \) is the area of the surface of the velocity discontinuity. Shear power for a half tooth is as follows:

\[ \sum W_s = W_s^{I+I} + W_s^{II+III} + W_s^{IV+V} + W_s^{V+VI} + W_s^{VI+VII} + W_s^{VII+II} \]

(29)

The friction power loss at material/punch or ejector is given by

\[ \dot{W}_f = \frac{m_a}{2 \eta} \int_0^L \left| \Delta V \right| dA \]

(30)

Where \( dA \) is the area of the die-material interface, \( m \) is the frictional constant and \( \left| \Delta V \right| \) is the velocity discontinuity at each interface. Frictional power for a half tooth is as follows:

\[ \sum W_f = W_f^{II+III} + W_f^{IV+V} + W_f^{V+VI} + W_f^{VI+VII} + W_f^{VII+II} \]

(31)

Detailed expressions for the internal, shear and frictional power are not given here because of their complexity. The total forging power for the half tooth, \( W_{total} \), is given by the summation of all the various powers derived from the six regions of deformation:
The total forging power for a spur gear with N teeth, $\bar{W}$, is given by

$$\bar{W} = \sum \bar{W}_i + \sum \bar{W}_s + \sum \bar{W}_f$$

(32)

The total forging power was minimized with respect to the bulge parameter $B$. It is seen that the energy terms derived from regions $I - IV$, are independent of parameter $B$. Hence, only powers dissipated in regions $V - VI$ were minimized. Average of the forging load, $F_{ave}$ is given by

$$F_{ave} = \frac{\bar{W}}{u}$$

(33)

Programs utilization

To study the application of the theoretical approach for spur gear forging, Al 2024 alloy was used for material. The flow stress of this material is expressed as $\bar{\sigma} = 358e^{0.155}MPa$ (Yang, 1994). Computer program for geometric modelling were written with Visual Basic in SolidWorks environment. Computer program for upper bound analysis were written using Matlab software. By running the computer program for geometric modelling user form shown in Figure 3 (a) will appear. Now user should input module, pressure angle, teeth number, gear height and bore radius of the spur gear for case study 1. With a click on the “Gear Modelling” command, solid model of gear is modelled as shown in Figure 3 (c). “Billet Design” command is for modelling of billet and with a
click on this command billet dimension is calculated and model of shown in Figure 3 (b). Model of die cavity, punch and ejector are shown in Figure 3 (d) (“Die Assembly” command).

User form for hollow spur gear is shown in Figure 4 (a) and solid model of gear, billet and die assembly are shown in Figure 4 (b) to (d) respectively.

We assume \( m = 0.1 \) for the friction factor. Figure 5 illustrates the comparison among the forging load (for solid spur gear) with including bulge (\( R \) is optimized in the computer program) and without bulging (\( R = 0 \)) and Choi and Chou (2000) experiment. As shown in this Figure the forging load predicted by this model (including bulge) is close to Choi and Cho experimental values.

CONCLUSIONS

Automatic modelling of the spur gear, design and modelling of billet and die cavity are one of the major capability of this computer program developed in this paper. Die cavity model can be input to a CAM software for machining.

The kinematically admissible velocity field in this study is very useful to predict the load requirement for the forging of spur gears.

The forging load increases with the increase of height reduction, friction factor and number of teeth. Forging load of the hollow billet is less than the solid billet.

![Figure 5. Comparison of forging loads between analysis and experiment [2] (hollow spur gear).](image1)

Figure 5. Comparison of forging loads between analysis and experiment [2] (hollow spur gear).

Figure 6 illustrates the variation of forging load for various values of bore radius for hollow spur gear. It is evident that forging load decreases with increasing in bore radius. This is because the area of the die-material interface decreases with increasing the bore radius, hence friction losses are lower. Also the volume of material will decrease and hence shear and internal power also is lower.

![Figure 6. Effect of bore radius on the forging load (hollow spur gear).](image2)

Figure 6. Effect of bore radius on the forging load (hollow spur gear).
The effect of the number of teeth on the variation of forging load for hollow spur gear is plotted in Figure 7. It is evident that forging load increases with increasing the number of teeth. This is due to the increased area of frictional surfaces and surfaces of velocity discontinuity as the number of teeth increases. Also the size of the gear increases with the number of teeth.

Figure 7. Effect of teeth number on the forging load (hollow spur gear).

REFERENCES