Impact of particle attributes on granular material

Amir Kalhor
Islamic Azad University, Shal branch, Department of civil engineering
corresponding author email: klhr_amir@yahoo.com

ABSTRACT: The characteristics of its particles have an important effect on the mechanical behavior of granular materials. Granular materials are treated as a continuum and its particulate nature is not explicitly considered. This paper establishes a new approach to determine index parameters which link the particle characteristics to the macro-scale response. The objective of research is to relate the micromechanical properties of the particles with the overall mechanical response of the material. We consider both of them. A new approach for the shape analysis of granular materials is proposed, and comparisons with classical methods are given. Modern techniques such as microscopy and interferometry are used. In a series of triaxial tests, shape and roughness have been found to affect the mechanical response of specimens of glass beads in terms of compressibility, stiffness and strength. There is more stick and slip for smooth particle specimens. In the last part of the paper a general discussion is reported and the main findings of the project are highlighted. Finally, some recommendations for further developments are given.

Key words: granular material, particle characteristics, the engineering behaviour

INTRODUCTION
Granular materials include soils and artificial substances constituted by particles not finer than powder grains, thus the lower limit of their grading is roughly 1 \( \mu \)m. Granular materials respond to applied loads in a highly complex manner. One mechanical response feature unique to granular materials is the sensitivity of the response to the intermediate principal stress. Sand is the most typical granular material. It results from the erosion of rocks and is globally widespread throughout alluvial lands, deserts, and the seabed. Engineers frequently face natural sands in their work. Improvements in the understanding of the relationship between the nature of particles and the overall response of natural sands are necessary for fundamental geotechnical applications (e.g. design of piled foundations, filters of large dams, pavements (Ernserberger & Eshelby, 1972&2006).

This paper makes a contribution to fundamental understanding of granular material response by examining the combined effects of friction and the relative magnitude of the intermediate principal stress on the material response. This study involved an extensive set of three-dimensional discrete element method (DEM) simulations using periodic boundaries (Feda, 1982). The first series of test simulations were triaxial compression tests on samples with equal packing density but differing coefficients of friction. Then the analysis was extended to true triaxial test simulations where the samples were subjected to axial compression at a constant mean stress for a range of intermediate stress ratios between 0 and 1.

The mechanical behaviour of sand and other granular materials depends on the characteristics of its particles. Typically we treat granular materials as a continuum and their particulate nature is not explicitly considered. Recent studies on the particulate mechanics of sand have focused on the behaviour of such soils at effective pressures higher than 1MPa. Although the response of sand to these conditions is of significance in some particular applications, such as oil-well stability and driven piles, in most of its applications the stresses involved are considerably lower. While there has been significant progress in recent years in understanding the impact of particle crushing on the response at high stress, less is known about the influence of particle characteristics on the mechanics of sand in the pressure range below 1MPa.

The paper outlines the simulation approach and presents the macro-scale, overall mechanical load-deformation response. Quantitative analysis of the material fabric using the contact normal force orientations and distributions allowed observations on the fundamental mechanisms underlying the observed response to be made (Fenton & Fonseca,2009&2008). The results indicate that the friction coefficient influences the inherent stability of
the strong force chains, while the intermediate stress ratio influences the lateral support provided to these force chains.

In particular, the global response of UGMs results in resilient and permanent deformations when subjected to repeated loading. Resilient deformations are related to the stiffness characteristics of the material that should be sufficiently high in order to avoid the fatigue cracking of overlying asphalt layers. On the other hand, the gradual accumulation of permanent deformations, although they are very small during each loading cycle, could lead to the collapse of the structure due to excessive rutting. Therefore, a well-designed pavement should experience accumulation of permanent deformations that during its service life will eventually cease resulting in a stable and basically resilient response (Ernsberger, 1972), (Friel, 1993) and (Frossard, 1979) in order to avoid premature failure.

This research is intended to fill this gap in the current knowledge. Both conventional and newly devised procedures have been used to characterize the geometrical and mechanical properties of granular materials. In particular, a micromechanical approach has been developed capable of characterizing a granular material's response at the particle

**Literature review**

This section deals with a literature survey of previous studies of the micromechanics of granular materials. This section focused on a review of other work on the macroscopic behaviour. As highlighted by ISO there are different methods and definitions for the descriptive and quantitative representation of particle shape. Ref (Gordon, 1976) clarified that "shape is taken to include every aspect of external morphology" and that the three independent aspects of shape are: form, roundness and surface texture. The adjective "independent" is important because it indicates the possibility of variation of each of these parameters without affecting the other two parameters. In his work Barret summarizes a number of different definitions of form and roundness proposed in literature. Barret's paper does not include any review of methods for measuring surface texture.

Particle size is a basic property used to characterize granular materials in engineering applications. Furthermore, definitions of representative sizes are important in describing the 3D geometry of a given particle (Grabko & Greenwood, 2002 & 1966). So, although it is not strictly a feature of shape, a discussion of size is an appropriate place to begin the consideration of morphological descriptions of particles.

If soil particles are dropped onto a horizontal plane, after scattering, they generally will lie with their smallest dimension perpendicular to that plane (Fig. 1). This orientation corresponds to the position of lowest potential energy of the particle on that plane, which was named by Feda (1982) the plain of greatest stability. The particle in Fig. 1 was generated using Matlab, it lies on the x–y plane and is enclosed by its smallest circumscribable cuboid (SCC) also lying on. The geometry of the SCC used here was defined as in Greenwood, 1976.

![Figure 1. The concept of smallest circumscribed cuboid SCC](image-url)
Figures. 1 (a) and (b) are two axonometric views of the particle inscribed in the SCC. These two views are from the top (a) and from the bottom (b). Referring to Fig. 1 (b), the simplest "cinematically consistent" geometric contact between the particle and is a triangular set of points, which, by definition, are the vertices of a triangle containing the vertical projection of the centre of gravity of the particle. The term "kinematically consistent" is used here to describe a stable particle position under the action of its self weight. This concept is generally ignored in the literature, but, as will be shown, it is of great relevance in the interpretation of the results of uniaxial compression tests on a single particle.

Figures. 1 (c), (d), and (e) are the three main projections of the particle. The dimensions of the three orthogonal axes of SCC are designated the Long (L), Intermediate (I) and Short (S) axis. These do not specify the particle uniquely, since an infinite number of differently shaped particles could be described within the same cuboid. However, for a given particle there can be only one cuboid. A simple rule for defining the geometry of this cuboid is given of this dissertation. None of the three cuboid axes necessarily coincides with either the maximum or minimum diameter of the particle. Similarly, the center of the cuboid does not represent the center of gravity of the particle, nor do the points of contact between the particle and the opposite faces of the cuboid lie. On a straight line that is parallel to an edge. However, in spite of these constraints, the dimensions L, I and S are a useful representation of particle size, and the axes of the corresponding cuboid are employed in the description of particle shape.

MATERIAL AND METHODS

A quantitative evaluation of grain shape is necessary in order to define parameters that can be inferred as index properties of a particulate material. With improvements both in image acquisition techniques and in image analysis algorithms, it is hoped that a quantitative description of particle geometry can be used to develop a classification framework for granular material. This tool would be analogous to the classical methods of clay classification proposed by Atterberg at the beginning of the last century and standardized later by (Greenwood, 2001). However, in contrast to the case of clay, in which the Atterberg limits are also pseudo-representative of the particle mineralogy, this proposed framework would consider geometric characterization only. Actually, for a defined grading, stress-history, porosity and water content, Atterberg's limits are indicators of important engineering properties of clays, such as permeability, shear strength and compressibility (Greenwood, 1942). In contrast, the estimation of the engineering behaviour of granular materials cannot be based solely on mere shape description of particles ever when the gradation, stress-history, porosity and water content and fabric of the particular soil are also known. This is because the mechanical properties, i.e. friction, resistance and deformability of single particles, as well as fabric, influence in different ways the permeability, shear strength and compressibility of granular materials (Hardin & Hardy, 1985 & 1925). Shape analyses of natural and artificial particles of the size of coarse sand was carried out using both manual devices and optical instruments (Table 1). The manual devices

<table>
<thead>
<tr>
<th>Technique/instrument</th>
<th>Type of measurements</th>
<th>d range (µm)</th>
<th>Feature of shape measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry sieving</td>
<td>Manual</td>
<td>75 - 4000</td>
<td>size</td>
</tr>
<tr>
<td>Micrometer</td>
<td>Manual</td>
<td>80 - 3000</td>
<td>size</td>
</tr>
<tr>
<td>Microscope</td>
<td>White light</td>
<td>10 - 7000</td>
<td>Size, sphericity, angularity</td>
</tr>
<tr>
<td>Mastersizer</td>
<td>Laser beam</td>
<td>0.4 - 900</td>
<td>size</td>
</tr>
<tr>
<td>Size &amp; shape analyser</td>
<td>Laser beam</td>
<td>10 - 3410</td>
<td>Size, sphericity, angularity</td>
</tr>
<tr>
<td>Interferometer</td>
<td>White light</td>
<td>10 - 100</td>
<td>roughness</td>
</tr>
</tbody>
</table>

Are straightforward but time consuming. Moreover, they generally interfere with the measured object and bias can affect the obtained data.

In contrast, optical instruments do not cause any mechanical disturbance to the measured object and allow the retrieval of a large number of measurements in a short time. The raw data are usually processed by software incorporated in the instrument. On the other hand, this category of tool suffers from limitations on the size and features of the processed particles. For example, in case of natural sand, the level of reactivity of the surface is a constraint for white light type of measurements (Horne, 1969). Additionally, they work with commercial prepackaged software, the performances and limitations of which are not always clear. A careful calibration is therefore necessary, as well as comparing different outputs from different apparatuses and manual devices (Horne, 1965).
Sieving

Sieving was performed in dry conditions on samples of material of at least 200g, using sieve meshes in the range of 75-4000m. The fraction retained on each sieve was weighed with the precision of 0.02g, and the cumulative distribution of the percentage passing was determined according to the usual standards (e.g. BS-410).

Particles interact via contact and non-contact forces. However, the non-contact forces (e.g. van der Waals and electrostatic forces) need only be considered for particles with the size of powders. Consequently contact interactions and resulting forces dominate response for the range of particles considered within the scope of this research.

The response at any contact depends on the shape and the mechanical properties of the two particles which to it belong. The proposal of a model for contact is one of the aims of this dissertation, and will be addressed in the next section on the basis of uniaxial micro-compression experiments carried out during the present research. However, the design of an apparatus for the uniaxial compression of coarse sand sized particles needs insight into predicting and recognizing the mechanical behaviour of the samples tested.

In this section, the contact response of an irregular particle of simple geometry, axially compressed between two hard smooth platens, is considered. Of simple geometry, axially compressed between two hard smooth platens, is considered.

Initial rotation

Figure 3 illustrates an irregular particle being compressed between two hard smooth horizontal platens. In order to simulate the experimental procedure of a compression test, the contact with the upper platen is assumed to occur after the particle was allowed to lie on its plane of maximum stability. Therefore, the two platens coincide with the two horizontal faces of the SCC of the given particle while no lateral contacts exist. Hence, at the initial stage of compression, the coordination number of the particle is \( N_c = 4 \) (i.e. three contacts of the irregular surface of the particle occur with the lower platen and one with the upper).

At this stage, the projection \( G' \) (Fig. 3) of the centre of gravity of the particle falls always inside the base-triangle CDE, \( (G'2CDE) \), the vertices of which coincide with the three points of contact with the lower platen. If also the vertical projection \( A' \) of the upper point of contact A falls inside the base-triangle CDE, \( (A'2CDE) \), the particle will have a stable reaction when the upper platen starts to push down on contact A (i.e. no rotation will be observed). In this case three vertical forces \( F_i \) will occur as reactions on the lower platen through the three points of contact C, D, and E (Fig. 3).

However, even if the initial stability criterion is met \( (G'2CDE) \), if the points \( A' \) and, for instance, E lie on the opposite sides of the edge CD \( (A'=2CDE) \), a rotation of the particle around an axis through C and D will occur as soon A is pushed down by the way in which, even a chemically description of contact-response, such as that described above occurs, depends on several geometric and mechanical parameters of the system considered, i.e. the shape of the particle, the coefficient of friction between the platen and particle, stiffness, crushing strength and the Weibull modulus of the material tested. These aspects will be analysed in the simulation developed of this section. However, before of this analysis is developed, a straightforward criterion for the assessment of the proneness of a given particle to initial rotation.

Figure 2. Micrometer for measuring sizes of coarse particles
upper platen. Schematically we can assume that this rotation develops until when the second highest point of the particle (B) comes into contact with the upper platen. For this condition to apply, the point A' and the projection B' of the point B must lie on the opposite sides of the rotation axis (CD) when the rotation is complete.

For the second, unstable scenario considered here, the nature of the early part of the inter-platen compression of an irregular particle experiencing initial rotation (A' = 2CDE) implies three consecutively different values of \( N_c \). In fact, the initial coordination number \( N_c = 4 \) reduces to 3 during the initial rotation, and afterwards increases again to 4 at the end of the rotation. At this final stage, only two points of contact with the lower platen (C and D), and two points of contact (A and B) with the upper platen occur. Starting from this situation, the full compression of the bulk can develop because the particle is clamped between two lines (i.e. C-D and A-B). These lines do not meet in 3D space.

They lie on the parallel planes defined by the loading platens and are both intersected by the vertical line of action of the compressive load. The coordination number remains 4, up to the point at which either fragmentation or crushing produces significant changes of the shape of the particle.

and the evaluation of the related displacement, can be proposed. The scheme in Fig. 4 is further simplified in Fig. 4. In this figure, two lateral views of an irregular particle, before (a), and after (b), the initial rotation, are shown. The particle considered is a hexahedron composed of two triangular pyramids, also known as triangular pyramid (Horne, 1965), which is the simplest solid having all its vertices coinciding with the five points governing the kinematics we wish to simulate. If \( A' \notin 2CDE \), the points A and B fall on the opposite sides of the vertical plane passing through the line CD, thus B is a second highest point, which will eventually stop the initial rotation. The
components of the maximum displacement of the apex A on the vertical plane can be related, respectively, to the minor diameter \( d_3 \) of the particle, the initial eccentricity of the apex \( e \), and the angle \( \beta = \alpha_0 - \omega \) (Fig. 4a). The angle \( \beta \), in turn, corresponds to the roundness of the particle at the corner considered (A). In fact, using Eq. 3.7, the relation \( RKS = 2 \text{wrtan}(\beta/2)/d_{\text{max}} \) holds.

The value of \( d_3 \) assumed here is larger than the smallest possible vertical diameter \( d_{F_{\text{min}V}} = d_0 \). However, if we consistently derive the three principal diameters of the particle using the method of the SCC introduced, we have to conclude that, in this case, \( d_3 \) does not coincide with \( d_{F_{\text{min}V}} \). Actually, the difference \( d_3 - d_{F_{\text{min}V}} = \delta \) characterizes the proneness to initial rotation of the particle, which occurs only if \( d_3 - d_{F_{\text{min}V}} > 0 \). From an examination of Fig. 5 we have:

\[
\begin{align*}
\delta v &= d_3(1 - \sin \beta/\sin \alpha_0) \\
\delta h &= d_3(\cos \beta/\sin \alpha_0 - 1/\tan \alpha_0)
\end{align*}
\]

where:

\[
\alpha_0 = \arctan \left( \frac{d_3}{e} \right)
\]

The value of the angle of friction \( \varnothing_{\text{mob}} \), mobilized during the compression, obeys the relationship:

\[
\arctan \frac{F_h}{F_v} = \varnothing_{\text{mob}} = \arctan(e/d_3) \leq \varnothing
\]

where \( \varnothing \) is the friction angle between the platen and particle. Hence, for any combination of a particle and a platen, a characteristic ratio, representing the lower limit for the occurrence of initial rotational instability, is given by the relationship:

\[
e/d_3 = \tan \varnothing
\]

For \( e/d_3 < \tan \varnothing \) any rotation during the test is restrained by the particle-platen friction. In this case, in spite of the progressive reduction of the inter-platen distance, and the favorable kinematical condition \( (e > 0 \text{ and } w > 0) \) rotation does not occur, and all the contacts stick frictionally against the platens. On the contrary, if \( e/d_3 > \tan \varnothing \text{ and } w > 0 \), the particle is not compressed at all during its rotational rearrangement between the platens.
RESULT AND DISCUSSION

It is useful to recall prior theoretical and numerical studies of the influence of surface roughness and friction on the mechanical response of granular materials. Ref (James,2000) gave formal proof of the stress- dilatancy equation introduced by (Janssens,2004) and showed how dilation influences the initial anisotropy observed in a triaxial test when rotund, rigid, smooth particles just start to move relative to each other, under an imposed deviator stress. Using a mathematical model, he found that the shear softening in triaxial compression is not dependent on the formation of slip lines but on the destruction of anisotropy due to dilation. He also pointed out that, when the assembly ultimately deforms at constant volume, a unique relationship between the angle of inter-particle friction and the angle of shearing resistance applies. Using an integrated theoretical and experimental approach applied to biaxial compression tests, a research came to a similar conclusion, but proposed the existence of slip lines, the extent of which they related to the inter-particle friction and the sizes of the aggregates. By means of DEM simulations [Haruyama & Johnson, 1969&1971]. Despite these theoretical analyses linking inter-particle friction to overall response, the experimental evidence of the influence of the surface roughness or inter-particle friction. On the overall shear characteristics of a granular material are still not well established. For instance, controlling the roughness of uniform steel spheres by means of rusting underwater erosion, and after triaxial tests.

The current study considers the response of glass balloting with controlled roughness and shape. These experiments have highlighted the influence of the texture on the initial density of the specimens and how this feature tends to cancel the demand of more frictional work by a rough surface. The same series of tests, coupled with results of compression tests on single particles of controlled roughness, have also confirmed the proposal that the internal properties of the particles, such as Young modulus and hardness, are the factors dominating the persistence of roughness under a shearing load. This section describes the results of these physical experiments. Some odometer tests on balloting are presented. A set of triaxial compression tests conducted on the same material is described and analysed. Summary table giving the characteristics of the materials considered as well as the overall response observed is provided. Finally, the conclusions of the macro-experimental work are drawn.

It is important to recognize that the macro-mechanical experiments carried out by the writer were supplemented by additional tests performed by MEng and MSc students working in consultation with the writer(James,2000).

FOG  Odometer test

In the current research 3 types of balloting were considered; as supplied balloting , etched balloting and crushed balloting. Each of these materials was subject to odometer tests at the room humidity. The etched balloting were produced by submerging the as supplied balloting in a water solution of hydrofluoric acid at 10% concentration for 1 hour. After this process, the roughness, RMSf, of the particles was in the range of 0.9-1.2 m.

The size of the three types of particle used was between 1:0 and 1:4mm. The specimens were prepared by air pluviating the balloting into a 38mm odometer ring from a funnel and the sample was vibrated to increase its density, where necessary. Typical odometer test data are shown in Fig. 6, and include the results of a first set of experiments carried out by Kwan, on balloting as supplied or etched, and Chan, on crushed balloting. These tests were conducted under a vertical stress which varied between 0.001 and 20MPa (Fig. 6).

In contrast to tests on the crushed balloting, where a well defined normal compression line (NCL) was reached, the yield of the as supplied or etched balloting was less evident.

To estimate the position of the NCL of the as supplied balloting, the writer performed additional tests achieving a maximum vertical stress of 44MPa, and the results of a typical compression test are shown in Fig. 6. Higher stress values could not be achieved in the available apparatus. Between 22 and 44 MPa the NCL was reached, and particle crushing was audible soon after the application of the load increment.

The end of the test stress and volumetric states are given. For the tests conducted at the cell humidity, or in a pseudo-dry manner, the accuracy in the determination of the volumetric strain was limited. In fact, the data obtained by the local and radial strain transducers were not precise at high strains limiting clear identification of the critical state volume of the sample. Despite these concerns, two of these tests are included for consideration here. As illustrated in Fig. 7, tests conducted at the relatively low confining pressure of 50kPa gave evidence of a slight tendency for both the peak and the final strength to increase with the initial sample density. However, it seems that each sample was still tending towards a critical state at the end of shearing (Fig. 7). In fact, a typical feature of these tests was the ability of the granular masses tested to support a fairly constant stress ratio even though a slight dilation was still occurring at the largest strains produced with the used apparatuses. Accepting that this constant stress ratio is assumed to be close to the critical state value, the tests in Fig. 7 show the in fluency of the roughness on the critical shearing resistance. Comparing the smooth and rough balloting, the increase in
roughness cancelled the effect of the higher initial porosity on the dilatancy. The milled sample had an initial void ratio of 0.629 but it experienced twice the volume triturated of the loosest sample of smooth balloting. As noted by Wu, stick-slip behaviour was rarely observed in rough samples at pre-peak stress level, whereas it was a typical feature in the tests on smooth balloting, whatever the stress.

Figure 6. Results of odometer tests; the test with highest vertical stress of 44MPa was carried out by the author, other tests were conducted.

Figure 7. Variations of stress and volumetric strain increasing the axial strain at 50kPa confining stress of a rough contact in an elastic system. In absence of plastic dissipation due to yielding of rough asperities, smooth particles store significant amount of elastic energy when compressed, and they can slip over each the
other if the rotation of particles can not occur at a given shear rate. Shi et al. found that the same ratio governs the localization of shear bands in a granular mass. They claimed that the lower the ratio $p2 \gamma = E2$ the higher the inhomogeneity in the shear strain within the assembly. The consistency of the two studies can be recognized if stick and slip is considered as an element of in homogeneity of the shear strain.

**CONCLUSION**

This dissertation has described an experimental study on the influence of the characteristics of individual particles on the overall mechanical response of coarse sand sized granular materials. The particle characteristics considered here can be classified as being geometrical (e.g. overall shape, surface topology) or mechanical (i.e. strength and stiffness of the bulk material).

Considering firstly the geometrical characterization, the shape analyses were conducted using modern techniques including optical microscopy, interferometry and shape analyzer.

The applicability of each of these tools to characterize particles was critically considered. The relationship between different shape proposed in earlier research was analyzed. Well established reference charts, whose use may be subjective, can be coupled with modern technologies to achieve objective measurements. A new indirect method (IMR) was introduced to quantify the roundness of coarse sand sized particles. The IMR exploits the fast process of the shape analysis performed by means of modern tools. It gives values of roundness that are consistent with the analytical method proposed by Wadell and adopted by several researchers in proposing the reference charts widely used to date. New criteria were proposed to decide at what scale roundness and roughness should be measured for characterization of a granular soil. The study of particle compression included a theoretical investigation of the mechanical response of an irregular particle compressed between two hard platens.

Interpretation of physical single particle compression tests should consider that irregular particles may rotate upon initial loading. The susceptibility to rotation depends on the shape of the particle and friction at the particle-platen interface. The roundness was recognized as a factor of proportionality between the normal stiffness and strength of the bulk material.

Experimental results were fitted by means of calibration of the most dominant parameters. This study proved the unavoidability of the shear of contacts of irregular particles whatever the increment of macroscopic stress.

**REFERENCE**