The Efficiency of Optimal Portfolio Selection Using Skewness Model

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ABSTRACT: One reason using of quantitative methods in investment management, this is development of financial economics. Financial economic development has expanded with portfolio optimization concept's. In fact, this is concept of portfolio optimization and diversification, the development of financial markets and financial decision-making. In this way, between of mathematical models and operational research in recent decades had been able to optimize affect portfolios. In this study, the classic portfolio optimization (Markowitz mean-variance model) in possible of using the skewed-based model as the objective function is investigated. In the present study is an attempt to model-based portfolio optimization using skewness (fuzzy) you can estimate and compare the risk and return portfolio risk and return anticipated this risk and return models predicted the results can be achieved in the classical model. In this study, we revision 195 monthly portfolio during 16 years and 3 months for listed companies in Tehran Stock Exchange and the risk and return of each portfolio based on two models of optimizational model was estimated skewness and classical optimization and in next step, using of the average difference could revision significant difference between risk and return were anticipated in the proposed methods. This study showed skewness model projected returns with returns predicted by the classical model, there are significant differences. And the predicted risk by the skewness model with risk predicted by the classical model, there are significant differences.

Key words: Risk, Return, Portfolio, Portfolio Optimization, Mean-Variance-Skewness, Fuzzy Programming, Fuzzy Triangular

INTRODUCTION

Investment development, on the one hand leads to inefficient investment and lead them to productive sectors of the economy and on the other hand, according to the orientation of people to invest in industries that profit will be directed more or less risk and finally lead to optimum allocation of resources. Select the optimal portfolio is one of the issues that engaged the minds of many capital market participants to have.

One of the challenges in today's world of investment return on assets, uncertainly with respect to future events and the consequences of them. The traditional approach to model this uncertainty, risk assets given as a random variable; but this approach the one hand, the imposition of unrealistic assumptions in selecting the optimal portfolio, on the other hand many problems in finding the probability distribution and the corresponding parameters will follow. What the different models apart from one another portfolio optimization, benchmark risk. Investment risk is one of the most important issues facing investors in stock markets. In general, investors are looking to earn maximum returns with minimum risk. So the concepts of portfolio optimization and diversification as a tool in the development and understanding of financial markets and financial decision-making have come. In this study, the efficiency of optimal portfolio selection model to evaluate and measure skewness, As well as the risk of the portfolio compared with the skewness model was selected portfolio risk using the classical model (model mean - variance). One of the ways to control investment risk, the optimal portfolio of stocks. Including methods of optimal portfolio selection model is based on the aim that in this study is skewed.
Theoretical foundations and overview of the literature

Behzadi, Adel Bakhtiarri and Mostafa, (2014) in presented research based on the average -Ntrvpy-skewness model is presented to solve the problem of portfolio optimization And to compare this model with the classic model of economic performance indicators are used. Finally, by using data from the Tehran Stock Exchange has been shown, the model is based on the average-Ntrvpy- skewed economic performance index higher.

Sabaaghian and ett.al. (2012), the concept of skewness defined as a third torque, and its properties are studied Mean-variance-skewness model and mean-variance model fuzzy, and the difference of change expressed. In order to solve the proposed model, the algorithm is designed to simulate fuzzy Mean-variance-skewness model is used to select the optimal portfolio.

Shah Mohamadi and et.al. (2012), in one study, the mean and variance and skewness portfolio selection based on the models considered, that in order to accommodate more models with the real world, return of stock for fuzzy variables are assumed. In this paper, to solve the non-linear model combines an intelligent algorithm to achieve optimal / near optimal offered. In this method, genetic algorithms and artificial neural network trained to search portfolios with fuzzy simulation used to estimate the portfolio's return and risk.

Raei and et.al. (2011), in connection with the portfolio optimization research results showed that: among the studied variables, including market risk premium, firm size, ratio of book value to market, market risk premium efficiency variable model skewness and kurtosis, and skewness size of the company, can better stock returns during the "-month conflict (April 2001 to Esfand 2005), in explaining the Tehran Stock Exchange And this model can help to optimize the portfolio.

Tehrani and et.al. (2009), in a study entitled “Effect of skewness and kurtosis in describing stock returns using the Capital Asset Pricing Model” the relationship between return and risk systematic skewness and kurtosis stock in Tehran Stock Exchange began the period 2002- 2006. Their sample included 11 companies listed on the Tehran Stock Exchange. The results pointed to the systematic risk and skewness an important role in describing stock returns in both periods play; however, over the period increased strain had a significant relationship with efficiency. However, in a period of decline, there is no significant relationship between stretching and efficiency.

Woo Chang Kim and et.al. (2014), during the investigation concluded that the classic method of computing the optimal portfolio selection problems ignored to fix the problem and suggested that the model, Markowitz mean-variance-skewness should be added. They have done research.

Ropak Batacharya and et.al. (2014), during the investigation concluded that by maximizing the asymmetry and to minimize the variance and cross-entropy, optimal portfolio selection is more reasonable. To solve the problem using these models, numerical examples, have gained for India BSE.

Brian Field et al (2012), Markowitz's portfolio composition, the mean and variance and skewness in investment performance extended. Their research shows that in addition to efficiency, liquidity is a concern for investors and every moment as a separate factor involved in investment performance. Concluded that the optimal extra portfolio to the skewness in synchronization with adding four first step required liquidity.

Burcu Aracioglu (2011), during the Istanbul Stock Exchange Istanbul an investigation concluded that return on a portfolio, at a time when skewed to the mean - variance Harry Markowitz added that the researchers will be significant changes, including: Kvanv (2003), Bana and et.al. (2007), Vtylan et al. (2010) and ... have done research in this field and to have reached the same conclusion.

Chiano and et.al. (2002), in this regard, the Taiwan study that such relationship between stock returns and skewness approved. They concluded that in periods of rising influence of skewness and kurtosis in describing stock returns to be more bearish periods.

Frengh and Lay (1997), they concluded that in terms of skewness and kurtosis upward impact on stock returns over the period of a further downgrade is described.

Howangh and Sachil, (1997), moments degrees higher returns in emerging markets were added to determine.

Mitra and Lowe (1997), skewness and kurtosis index of emerging markets compared with developed markets. However, many of the studies, in this context, the emerging markets have been on the market index very few studies have been conducted and on the stock.

Kervas and Litzenberg, (1976), CAPM model developed by adding skewness and concluded that the addition of skewness increased explanatory power of the model.

Francis (1995), showed that the skewness of the stock does not affect pricing. However, Kraus and Lytzenbrgr (1976), stated that systematic skewness (the skewness) in stock pricing is important. The most important issue for investors, particularly in economic activity, the issue of how to allocate capital to one or a few different options to provide maximum return on investment, and incur less risk. This study proposes an easy way to support
the decision maker in choosing a suitable investor's portfolio. In this study, mean-variance portfolio selection model based on asymmetry or skewness (phase model) is considered, the more models to adapt to the real world, stock returns are assumed to be fuzzy variables.

In this study, credibility theory is used to calculate the required parameters. To compare the mean-variance model and the economic is used performance index. Finally, using data from the Tehran Stock Exchange has been shown, the model based on mean, variance, skewness phase, the economic performance index higher. And research question is: Is optimize portfolio risk in a classic-risk portfolio optimization model is skewed significant difference?

Spanish portfolio in simple terms refers to a combination of assets that are held by an investor to invest. This investor could be individual or a legal entity. Technically, a portfolio includes a set of real estate and financial investment is an investment. However, in this study the emphasis is on financial assets. Study all aspects of the portfolio, the portfolio is managed. This general term encompasses the concepts of portfolio theory. In 1952, Harry Markowitz portfolio to provide basic model that was the basis for modern portfolio theory. Before Markowitz investors were familiar with the concepts of risk and return. Although they were familiar with the concept of risk, but usually could not measure it. However Markowitz was the first to be expressed in terms of portfolio and diversify the scientific method. He showed quantitatively why and how portfolio diversification can reduce the risk of an investor's investment.

Why it is important for investors to diversify investment? We can say that the number one portfolio management is diversification. Because investors are uncertain about the future need to reduce the risk to diversify their investments. In other words, the formation of a diversified portfolio, greatly reduces the risk. For example, in the economic crisis America 1987, Fqtkmtraz % Mutual Funds (Who had formed the portfolio) were faced with losses.

Markowitz decided to make methods and ideas available in the form of a formal framework to organize and respond to the fundamental question: Is the risk of individual securities with a total risk of the portfolio, which is comprised of a total portfolio? Markowitz presented models of portfolio risk measurement methods to calculate the expected return on portfolio risk and pay. His model is based on the expected return and risk characteristics of securities that theoretical framework for analyzing risk and return options, it is stable.

As well as lack of knowledge and proper use of theoretical frameworks portfolio selection, always has trouble investors. Iran's capital market is no exception. Insufficient knowledge of portfolio selection procedures and a lack of awareness of the role of quantitative methods in this field, many market participants is difficult. With the development of capital markets and the dramatic growth of hedge funds in recent years, he has found more ways to replace it. In the present study the models and methods have been adapted, the model of this study will be used in portfolio selection problems in Iran.

The role of optimal portfolio decisions and risk investors and capital market participants is obvious. Skewness model including a model that could optimize an important role in portfolio selection and investment decisions in the context of the play. This research is aimed to be an optimal portfolio selection model skewness in the study and review.

There are tools and scale to assess, review, and submit such a model in the country was due to the effect of skewness in the optimal portfolio selection model to understand and cover and to the development of practical management and skewness in portfolio selection model that is intended to reduce risk and improve decision making investors. The findings of this study can be enabled for all managers of investment companies on the stock exchange, shareholders and institutions active and strategic use of the investment managers.

Dimensions and variables influencing the choice of an investor's portfolio approach at the macro level And how to model these variables in a mathematical model of the main objectives of this research is considered. In addition, the characteristics of the model, it has the advantage of not restricting it to a group of investors, all investors can research findings that Iran is considering the specific circumstances can useing of the capital market. According to the new definition of skewness model optimization this template can Iran and establish more facts on the capital market, especially in investment funds provide portfolio. Due to the introduction of a variable rate of return as well as interaction with risk capital market and money market realized. This research is mathematical model investor's portfolio choice in terms of skewness model assumptions, therefore, in terms of goal, and from the standpoint of running a descriptive research with applied research methods and comparative correlation. Which uses mathematical techniques and modeling is done.
RESEARCH’S METHODS

Regarding the purpose of the present study is correlational research. In this study, using the achievements of modern information software related to the mining companies listed on the stock exchange And by using Excel program was used to classify data. After using model-based optimization model skewness and classic, information about the companies listed in Tehran Stock Exchange using the new software version has arrived outcomes, portfolio optimization based on the framework of skewness and classic, action is taken, and the next step is to compare the predicted risk and return both methods will be discussed. The extracted information of one year of exchange listed companies Tehran a year of the application period Rah Avard Novin including 195 in the period from 2010 to 2016 was observed using Excel program for each year based on the ratio of the average daily value of transactions they participate in the total value of the transactions was discussed. Then, using the standard deviation, variance, covariance matrix table form With average returns by Lingo software to optimize the portfolio in two ways: 1. skewness model-based method. 2. classic "mean - variance of Markowitz," these companies were using the average return and risk. The output of these two optimization methods to calculate the weight of each asset in the portfolio composition and risk and return portfolio is anticipated. At the end of comparing the significance of these two optimization methods to determine the effectiveness of the proposed method in this study, by statistical analysis software Spss, was put to discussion and analysis.

Research Hypothesis
Portfolio risk prediction model based on skewed significantly different at the 90% level of portfolio risk prediction is based on classical optimization.
Portfolio risk prediction model based on skewed significantly different at 95% portfolio risk prediction is based on classical optimization.
Portfolio risk prediction model based on skewed significantly different at 99% portfolio risk prediction is based on classical optimization.

Models
Markowitz's Mean-Variance Model Of Portfolio
The definition of Markowitz mean-variance model for portfolio formation is as follows (Markowitz, 1952):

The variable N represents the number of shares investors that they can be top stocks chose to Portfolio . This investment shares and the set A = {a\textsuperscript{1}, ..., an} are shown. For each share i, Ai ID is intended, This variable is the same identification code is assigned by the Exchange per share; the expected return and risk Ri's share of ai per share price is shown with variable variance. It is possible to have such shares to each dependent variable cij are used to show the correlation between stocks. I variable wi represents the weight for each share of share i in the portfolio. Each share in the company's portfolio of non-negative weight so that the total weight of the stock is equal to relationship and portfolio P = \{w\textsuperscript{1}, ..., wn\} is a collection of stocks that wi weight per share in the portfolio.

\[ \sum_{i=1}^{n} Wi = 1 \]

To obtain the return of any deal is done, changes compared with last closing price of the last stock prices previously it used to be means for calculating the return on the transaction at time t and \( t-1 \) is as follows:

\[ r_t = \frac{(P_t - P_{t-1})}{P_{t-1}} \]

\( P_t \): The latest share price closed at time t,
\( P_{t-1} \): The last closing price of the share at the time of \( t-1 \),
\( R_t \): Return of the equation at time t.

Stock price changes can cause the growth and profitability of its portfolio or cause losses. Just as changes to investment risk arises, according to the definition of variance model, Risk amount per share equal to the amount of variance in the prices of transactions that occur in a period expressed:

\[ \sigma^2 = \frac{1}{M - 1} \sum_{i=0}^{n} (P_i - \eta)(P_i - \bar{P}) \]

In the above equation is calculated range prices for M day.

Variable \( \eta \): average yield per transaction compared to the previous transaction in the course of trade;
Variable \( \bar{P} \): average stock price during this period;
\( P_t \): The last closing price of the share on the day i
After the above calculations, can be assessed and based on this evaluation and stock specific target function, selected stock to form the portfolio selection. After selection and determine the relative weight of each stock in the portfolio, can be used again by the mean-variance model, returns and portfolio risk can be calculated. Portfolio returns the sum of the weighted average return per share is obtained, which is expressed in the following equation:

\[ E(R_p) = \sum_{i=0}^{n} W_i \times E(R_p) \]

Variable \( E(\text{Rp}) \): the expected return of the portfolio;
Wi: Weight per share in the portfolio is determined by algorithm;
Variable \( E(\text{Ri}) \): \( i \) is the share of expected returns;
Variable \( n \): number of shares of the Portfolio.

With shares in portfolio may have direct or indirect relationship with each other. This relationship is expressed by the correlation coefficient to calculate portfolio risk stock used is as follows:

\[ \sigma_p^2 = \sum_{j=1}^{n} \sum_{i=1}^{n} w_i w_j \sigma_{ij} + \sum_{i=1}^{n} w_i^2 \sigma_i^2 \]

The correlation coefficient between two stocks:

\[ \sigma_{k,j} = \sum_{j=1}^{n} [R_{ki} - E(R_k)] [R_{ji} - E(R_j)] P_i \]

Variable \( \sigma_{k,j} \): covariance i and j stock show,
Variable \( P_i \): probability of occurrence of each of the scenarios.
This value is considered the same for everyone. One of the possible returns in the period \( k \) \( P_k \) variable share transactions. These relations are based on the Markowitz model is proposed. Markowitz model is a mathematical function as follows:

\[ \text{Max} \sum_{i=1}^{m} R_i X_i \]

\[ \text{s. t } \text{Min} \sum_{i=1}^{m} \sum_{j=1}^{m} \sigma_{ij} X_i X_j \]

\[ X_i \geq 0 \]

In this equation:
m: number of assets in the portfolio;
\( X_i \): the proportion of capital that can be invested in asset i;
\( R_i \): expected return on asset i;
\( \sigma_{ij} \): covariance between the return on assets i and j;
\( Xi \geq 0 \): implied that short selling.

Return portfolio, is equal to the sum of the weighted return on all assets included in the portfolio and the weight applied to each asset returns, a proportion of the investments mentioned in assets. Risk of the portfolio is calculated based on the covariance between the return on assets in the portfolio.

**Skewness**

Skewness refers to the asymmetric distribution. The asymmetry in the right tail of a distribution that has been extended, , The positive skewness or skew to the right is famous . While a distribution tail asymmetry that developed on the left side, being famous to skew or skew to the left is known. Skewness can accept negative infinity to positive infinity value. The asymmetry of the probability distribution is skewed, indicating that deviations from symmetry is one of the items. If you are symmetric with respect to the mean, skewness is zero, in which case the mode, median and mean are the same.

Skewness of the third torque is normalized. Skewness in the distribution function is a measure of the presence or absence asymmetry.

For a perfectly symmetrical distribution of zero skewness and kurtosis for an asymmetric distribution with positive skewness towards higher values and for asymmetrical distribution with negative skewness value is stretching toward smaller quantities.
Several formulas to calculate the coefficient of skewness is provided but in this study, the following formula is used to calculate the coefficient of skewness of the portfolio to calculate the coefficient of skewness is famous moments ways:

\[ sk = \frac{r_3}{\sigma_p^3} \]

\[ r_3 = \frac{\sum F_i (X_i - \mu)^3}{N} \]

Sk coefficient of skewness, pσ SD, Fi frequency, Xi weight, iμ average, N number

**Mean-variance model skewness**

The main objective of this research-based portfolio optimization model is skewed And not because of the uncertainty of the data and the accuracy of the concepts of fuzzy logic has been used to solve the problem. The objective function to solve the problem is to max skew of which is shown as follows:

\[ \text{Max} \sum_{i=1}^{n} r_i w_i \]

\[ \text{Min} \sum_{i=1}^{n} p(l_i + s_i) \]

\[ \text{Min} \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij} \]

\[ \text{MaxE}(\frac{\mu^+(r_3-I)}{\sigma})^3 \]

**Fuzzy logic**

Fuzzy theory by Professor Lotfi Zadeh in 1965 in an article called fuzzy sets was introduced. One of the most important parts of fuzzy theory is faced with many real-world phenomena in which there is uncertainty is used, it is theoretically possible. This theory, probability theory is somewhat similar, with the difference that in probability theory based on the occurrence of an event that is in the past While in theory allows the possibility of an accident in addition to statistical studies about the possibility of the event is logically depends. While the possibility of a fuzzy event, very important and widely used fact. However, this duality is not your property. Having this property will lead to the greatest extent possible occurrence of an event decisive phase can ensure this event as a result of the setting up of Nmytvtnl to the value of confidence. In 2002, Li and Liu theory of fuzzy sets reputation as an option for a possible rival, offered. Rate this duality is a measure of your property. Therefore, after the presentation of the basic concepts of reliability theory has been expanded rapidly. To understand more material presented in this research are required review of facts and theory will be credited with fuzzy variables.

**Triangular fuzzy numbers (TFN)**

Triangular fuzzy number (TFN) is a fuzzy number with three real number as \( F = (l, m, u) \) is displayed. Bound up with u fuzzy number F can be shown that has the highest values. The lower bound is shown that it has the minimum amount that can be fuzzy number F. M is the most probable value of a fuzzy number. The membership function is a triangular fuzzy number as follows:

\[ \mu_f(x) = \begin{cases} 
 x - l & l < x < m \\
 m - l & l < x < m \\
 u - x & l < x < m \\
 u - m & \text{other ways}
\end{cases} \]

Triangular fuzzy number \( F = (l, m, u) \) in the geometric space as shown below:
**Triangular fuzzy number**

According to the triangle membership function if \( x \) is between \( l \) and \( m \) is greater then ever. It will be far greater degree of membership for \( x = m \) degree of membership is equal to one. If \( x \) is between \( m \) and \( u \) is the bigger, the smaller will be the membership and the membership will be zero at \( x = u \).

**Algebraic operations on triangular fuzzy numbers**

Computational efficiency due to the triangular fuzzy numbers simply do math operations on it very much. Mathematical operations on fuzzy numbers such as \( F_1 \) and \( F_2 \) can be done by simply:

\[
\begin{align*}
F_1 &= (l_1, m_1, u_1) \\
F_2 &= (l_2, m_2, u_2) \\
F_1 \oplus F_2 &= (l_1 \oplus l_2, m_1 \oplus m_2, u_1 \oplus u_2) \\
F_1 \ominus F_2 &= (l_1 \ominus l_2, m_1 \ominus m_2, u_1 \ominus u_2) \\
F_1 \otimes F_2 &= (l_1 \otimes l_2, m_1 \otimes m_2, u_1 \otimes u_2) \\
F_1^{-1} &= \left( \frac{1}{l_1}, \frac{1}{m_1}, \frac{1}{u_1} \right)
\end{align*}
\]

**Fuzzy variable**

Suppose \( \xi \) is fuzzy variables and stock return \( i \) will show and let \( x_i \) is the proportion of capital that can be invested in stocks, in general, \( \xi \) Madle. \( F_i \) / \( F_i + di \) is taken where \( P_i \) is the current price of securities, \( P_i \) stock price forecast for next year and \( di \) estimation. The amount of revenue bonds during the next year. When least expected returns and have the highest risk, investor using skewness, portfolio would prefer to have a larger skewness. We Mean-Variance model, we propose skewness.

**Fuzzy variable skewness**

To solve the problem using the Fuzzy variable skewness as a way to simplify application is used, in which case the variable skewness, skewness called fuzzy.

In this section, the skewness for the variable phase we define and discuss the properties of its base.

Definition 2.1 Suppose \( \xi \) fuzzy variable with returns expected to be limited. Skewness is defined as \( \xi \),

\[
S[\xi] = E[(\xi - E[\xi])^3] 
\]

Example 2-6. Let \( \xi = (a, b, c) \) is triangular fuzzy variables, then the following result is easily obtained.

\[
S[\xi] = \frac{32}{32}[(c - b) - (b - a)]
\]

Suppose \( \xi \) fuzzy normally distributed variable with density function \( \mu (x) = 2 \left[ 1 + \exp \left( \frac{\pi | xe |}{\sqrt{\delta \sigma}} \right) \right] (-1) \) For each real number \( r \) credibility inversion theory result That:

\[
Cr[\xi \leq r] = \left( 1 + \exp \left( \frac{\pi (e - r)}{\sqrt{\delta \sigma}} \right) \right)^{-1}
\]

\[
Cr[\xi \geq r] = \left( 1 + \exp \left( \frac{\pi (r - e)}{\sqrt{\delta \sigma}} \right) \right)^{-1}
\]

And the definition will have 2-1:

\[
S[\xi] = E[(\xi - E[\xi])^3] = \int_0^{+\infty} Cr[(\xi - e)^3 \geq r] dr - \int_0^{-\infty} Cr[(\xi - e)^3 \leq r] dr
\]

\[
= 3 \int_0^{+\infty} Cr[(\xi - e)^3 \leq r] dr = 3 \int_0^{+\infty} r^2 Cr[\xi \leq r + e] dr
\]

\[
= 3 \int_0^{+\infty} r^2 \left( 1 + \exp \left( \frac{\pi r}{\sqrt{\delta \sigma}} \right) \right)^{-1} dr = 3 \int_0^{+\infty} r^2 \left( 1 + \exp \left( -\frac{\pi r}{\sqrt{\delta \sigma}} \right) \right)^{-1} dr
\]

\[
= 3 \int_0^{+\infty} r^2 \left( 1 + \exp \left( \frac{\pi r}{\sqrt{\delta \sigma}} \right) \right)^{-1} - r^2 \left( 1 + \exp \left( \frac{\pi r}{\sqrt{\delta \sigma}} \right) \right)^{-1} dr = 0
\]

Suppose \( \xi \) fuzzy variable with limited expected return. For any real number \( a \) and \( b \), we have:
\[ S[\alpha \xi + b] = a^3 S[\xi] \]

**Fuzzy Distribution**

It is easy to show that: \( E[\alpha \xi + b] = aE[\xi] \)

\( S[a] = E[(\alpha \xi + b - (aE[\xi] + b))^3] = a^3 E[(\xi - E[\xi])^3] = a^3 S[\xi]. \)

Suppose \( \xi \) variable symmetric fuzzy with limited expected return, then we have:

\[ S[\xi] = 0 \]

Knydμ function \( \xi \) is supposed to represent. Since \( \xi \) is symmetric e is a real number such that:

\( \mu(e + r) = \mu(e - r), \quad \forall r \in R. \)

In addition achieved that:

\[ \text{SUP} \frac{S \geq r}{\text{SUP}} e^\mu(S) = \text{SUP} S \geq r + e^\mu(S + e) = \text{SUP} S \geq r + e^\mu(e - S) = \text{SUP} S \geq r + e^\mu(S). \]

The theory will have credibility inversion:

\[ Cr(\xi \geq r + e) = \frac{1}{2} \left( \text{SUP} S \geq r + e^\mu(S) + 1 - \text{SUP} S \geq r + e^\mu(S) \right) = \frac{1}{2} \left( \text{SUP} S \leq e + x^\mu(r) + 1 - \text{SUP} S \geq e + x^\mu(r) \right) = Cr(\xi \leq e + x) \]

First show \( E[\xi] = e \) In fact, according to the definition of expected returns

\[ E[\xi] = \int_0^{\infty} Cr(\xi \geq r)dr - \int_0^{-\infty} Cr(\xi \leq r)dr = \int_0^{\infty} Cr(\xi \geq r + e)dr - \int_0^{-\infty} Cr(\xi \leq r)dr \]

\[ = \int_0^{\infty} Cr(\xi \geq r + e)dr + \int_0^{-\infty} Cr(\xi \leq r + e)dr - \int_0^{\infty} Cr(\xi \leq r + e)dr - \int_0^{-\infty} Cr(\xi \geq r + e)dr = \int_0^{\infty} Cr(\xi \geq r)dr - \int_0^{\infty} Cr(\xi \leq r)dr = e \]

In addition, the definition of skewness is obtained that:

\[ E[(\xi - e)^3 \geq r]dr = \int_0^{\infty} 3r^2 Cr(\xi - e \geq r)dr - \int_0^{-\infty} 3r^2 Cr(\xi - e \leq r)dr = \int_0^{\infty} 3r^2 Cr(\xi - e - r)dr - \int_0^{-\infty} 3r^2 Cr(\xi - e - r)dr = 0 \]

It was completed (Sabaghian Toosi, 2012).

**Fuzzy skewness mean-variance model**

Suppose \( \xi \), fuzzy variable i shows that stock returns and assuming xi is the proportion of capital that can be invested in securities \( \xi \) is generally equivalent to \( (P_{-i} + d_i - P_{-i}) / P_{-i} \) taken where pi is the current price of securities. \( P_{-i} \) price forecast for next year is estimated Securities and di. The amount of future income securities during a year when least expected returns a and γ greatest risk are given, investor using skewness, skewness larger portfolio that is preferred. So we model the mean-variance, skewness above we recommend.
\[
\begin{align*}
\text{maximize} & \quad S[\xi_1X_1 + \xi_2X_2 + \cdots + \xi_nX_n], \\
\text{Subject to:} & \quad E[\xi_1X_1 + \xi_2X_2 + \cdots + \xi_nX_n] \geq \alpha \\
& \quad V[\xi_1X_1 + \xi_2X_2 + \cdots + \xi_nX_n] \leq \gamma \\
& \quad X_1 + X_2 + \cdots + X_n = 1, \\
& \quad X_i \geq 0, \quad i = 1, 2, \ldots, n.
\end{align*}
\]

Indicating the first states that the expected return would not fall below a specific threshold indicating \( \alpha \) and the second states that the risk will be lower than the specified limit. The last two states indicating that investors all over the \( n \) securities investment funds and that can not be negative value of an asset. Suppose \( \xi_i = (a_i, b_i, c_i) \) independent triangular fuzzy variables for \( i = 1, 2, \ldots, n \) is specified then the model will become the definitive application

\[
\begin{align*}
\text{max} & \quad \left( \sum_{i=1}^{n} x_i(c_i - a_i) \right)^2 \left( \sum_{i=1}^{n} x_i(c_i - a_i) \right), \\
s.t & \quad \sum_{i=1}^{n} x_i(a_i + 2b_i + c_i) \geq 4\alpha, \\
& \quad 11 \left( \sum_{i=1}^{n} x_i(c_i - a_i) \right)^2 \left( \sum_{i=1}^{n} x_i(2b_i - a_i - c_i) \right), \\
& \quad +2\left( 8 \sum_{i=1}^{n} x_i(c_i - a_i) + 3 \sum_{i=1}^{n} x_i(2b_i - a_i - c_i) \right) \left( \sum_{i=1}^{n} x_i(c_i - a_i) \right)^2 + \left( \sum_{i=1}^{n} x_i(c_i - a_i) \right)^2 \\
& \quad \leq 192\gamma \left( \sum_{i=1}^{n} x_i(c_i - a_i) + \sum_{i=1}^{n} x_i(2b_i - c_i - a_i) \right) \\
& \quad x_1 + x_2 + \cdots + x_n = 1, \\
& \quad x_i \geq 0 \quad i = 1, 2, \ldots, n
\end{align*}
\]

Proof: Since the \( \xi_i = (a_i, b_i, c_i) \) triangular fuzzy variables are fully developed originally developed by Lotfi Zadeh in this way, we find that:

\[
\sum_{i=1}^{n} \xi_i X_i = \left( \sum_{i=1}^{n} x_i a_i, \sum_{i=1}^{n} x_i b_i, \sum_{i=1}^{n} x_i c_i \right).
\]

This will be the triangular fuzzy variables, plus we have:

\[
E[X_1 \xi_1 + X_2 \xi_2 + \cdots + X_n \xi_n] = \frac{\sum_{i=1}^{n} x_i(a_i + 2b_i + c_i)}{4}
\]

In addition, we have:

\[
V[X_1 \xi_1 + X_2 \xi_2 + \cdots + X_n \xi_n] = \frac{11\left(\sum_{i=1}^{n} x_i(c_i - a_i)\right)^2\left(\sum_{i=1}^{n} x_i(2b_i - a_i - c_i)\right)}{192\left(\sum_{i=1}^{n} x_i(c_i - a_i) + \sum_{i=1}^{n} x_i(2b_i - a_i - c_i)\right)}
\]

\[
+ \frac{2\left(8\sum_{i=1}^{n} x_i(c_i - a_i) + 3\sum_{i=1}^{n} x_i(2b_i - a_i - c_i)\right)\left(\sum_{i=1}^{n} x_i(c_i - a_i)\right)^2 + \left(\sum_{i=1}^{n} x_i(c_i - a_i)\right)^2}{192\left(\sum_{i=1}^{n} x_i(c_i - a_i) + \sum_{i=1}^{n} x_i(2b_i - a_i - c_i)\right)}
\]

By substituting the equations in the model proved to be the case

**The findings**

**The correlation between the variables**

Correlation coefficients expressing the relationship between the variables in other words, correlation analysis of statistical tools to determine the type and degree of relationship between variables.

In this study, to determine correlations between variables Spearman's correlation coefficient was used because if the number of data not reasonably low and the assumption of normal Spearman's correlation coefficient
was used, based on the original amount of data is calculated based on the rating mark it (rs) between +1 and -1 is show and always.

<table>
<thead>
<tr>
<th>Variables</th>
<th>RISK%90</th>
<th>RISK%95</th>
<th>RISK%99</th>
<th>markoeitz Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>RISK</td>
<td>1.00</td>
<td>0.627**</td>
<td>0.609**</td>
<td>-0.088</td>
</tr>
<tr>
<td>%90</td>
<td></td>
<td>0.000</td>
<td>0.000</td>
<td>0.977</td>
</tr>
<tr>
<td>RISK</td>
<td>0.627**</td>
<td>1.00</td>
<td>0.069**</td>
<td>-0.139</td>
</tr>
<tr>
<td>%95</td>
<td></td>
<td>0.000</td>
<td>0.000</td>
<td>0.053</td>
</tr>
<tr>
<td>RISK</td>
<td>0.609**</td>
<td>0.690</td>
<td>1.00</td>
<td>-0.082</td>
</tr>
<tr>
<td>%99</td>
<td></td>
<td>0.000</td>
<td></td>
<td>0.252</td>
</tr>
<tr>
<td>N</td>
<td>195</td>
<td>195</td>
<td>195</td>
<td>195</td>
</tr>
<tr>
<td>Coefficients</td>
<td>-0.088</td>
<td>-0.0139</td>
<td>-0.082</td>
<td>1.000</td>
</tr>
<tr>
<td>Significance</td>
<td>0.219</td>
<td>0.053</td>
<td>0.252</td>
<td></td>
</tr>
</tbody>
</table>

If the data is not relative interval can convert them into place. No matter what the dependent variable and one independent variable. If a correlation coefficient of zero indicates no correlation, If the correlation coefficient is less than zero, the negative correlation is imperfect and that is another variable decreases with the increase, if partial correlation coefficient is greater than zero and is positive that with an increase in one variable, the other increases and if zero indicates no correlation. According to the above correlation matrices between quantitative variables are presented in the table above.

Not analyzed due to lack of significant coefficients which are as high table.

**The results of tests are**

Portfolio risk prediction model based on the skewness in 90%, 95% and 99% of predicted risk portfolio optimization based on classic are not normally distributed the Wilcoxon test was used to test hypotheses of this test, non-parametric statistical tests. To assess the similarity of two related samples ordinal scale is used. According to this hypothesis using the above test is as follows. Then, according to the test the difference between the two population expected portfolio risk Skewness and risk-based model portfolios predicted by classical optimization to provide the following assumptions:

**The results of the first sub-hypothesis about risk**

First hypothesis: the risk predicted by the model portfolio at the level of 90% based on skewed significantly different from the predicted risk portfolio is based on classical optimization.

\[ H_0: \mu_1 \geq \mu_2 \]
\[ H_1: \mu_1 < \mu_2 \]

1\( \mu \): portfolio risk prediction model based on the skewness at the level of 90%

2\( \mu \): risk predicted by classical optimization portfolio

<table>
<thead>
<tr>
<th>D.F</th>
<th>Sig.</th>
<th>Z</th>
<th>The average risk of classical optimization</th>
<th>The mean risk-based skewness at 90%</th>
<th>Risk of classical optimization</th>
<th>Total risk-based skewness at 90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2491</td>
<td>0.00</td>
<td>-11.110</td>
<td>0.0014</td>
<td>0.2505</td>
<td>195</td>
<td>195</td>
</tr>
</tbody>
</table>

As predicted in the table above risk-return portfolio optimization based on the classic lower than predicted portfolio risk-based model is skewed at 90% the difference between the two communities is also due to the fact that the statistic z, 110 / 11- under 5% significance level . It can be anticipated that the portfolio risk-based model skewness at the level of 90% Significantly different from the predicted risk portfolio is based on classical optimization It is also noteworthy that this difference is significant or superiority.

**The results of the second sub-hypothesis about risk**

The second sub-hypothesis: portfolio risk prediction model based on skewed significantly different at 95% predicted risk portfolio is based on classical optimization.
\[ H_0: \mu_1 > \mu_2 \]
\[ H_1: \mu_1 \leq \mu_2 \]

1μ: risk prediction model based on the skewness of the portfolio at 95%
2μ: risk predicted by classical optimization portfolio

Table 3. Wilcoxon test results of the second sub-hypothesis about risk

<table>
<thead>
<tr>
<th>D.F</th>
<th>Sig.</th>
<th>Z</th>
<th>The average risk of classical optimization</th>
<th>The mean risk-based skewness at 95%</th>
<th>Risk of classical optimization</th>
<th>Total risk-based skewness at 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2489</td>
<td>0.00</td>
<td>-10.854</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mean</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0016</td>
<td>195</td>
</tr>
<tr>
<td>0.2505</td>
<td>195</td>
</tr>
</tbody>
</table>

As predicted in the table above risk-return portfolio optimization based on the classic lower than predicted portfolio risk of the model is based on the skewness at 95% The difference between the two communities is also due to the fact that the statistic z, 854 / 10- under 5% significance level It can be anticipated that the portfolio risk based on the predicted risk skewed significantly different at 95% of the portfolio is based on classical optimization It is also noteworthy that this difference is significant or superior.

The third sub-hypothesis test results about risk

The third sub-hypothesis: the risk prediction model based on the skewness of the portfolio at 99%, significantly different from the predicted risk portfolio is based on classical optimization.
\[ H_0: \mu_1 > \mu_2 \]
\[ H_1: \mu_1 \leq \mu_2 \]

1μ: risk prediction model based on the skewness of the portfolio at 99%
2μ: risk predicted by classical optimization portfolio

Table 4. The results of the Wilcoxon test for third sub-hypothesis about risk

<table>
<thead>
<tr>
<th>D.F</th>
<th>Sig.</th>
<th>Z</th>
<th>The average risk of classical optimization</th>
<th>The mean risk-based skewness at 99%</th>
<th>Risk of classical optimization</th>
<th>Total risk-based skewness at 99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2463</td>
<td>0.00</td>
<td>-10.874</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mean</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0042</td>
<td>195</td>
</tr>
<tr>
<td>0.2505</td>
<td>195</td>
</tr>
</tbody>
</table>

As predicted in the table above risk-return portfolio optimization based on classical less than predicted risk Skewness model portfolio based on the level of 99%. So there are differences between the two communities Also, given that the statistic z, 874 / 10- under 5% significance level It can be anticipated that the portfolio risk-based model skewness significant difference at 99% portfolio risk predicted by classical optimization is also noteworthy that this difference is significant or superiority.

CONCLUSION

The average predicted risk portfolio optimization based on the classic lower than predicted portfolio risk-based model is skewed at 90%.
The average predicted risk portfolio optimization based on the classic lower than predicted portfolio risk-based model skewness is at 95%.
The average predicted risk portfolio optimization based on the classic lower than predicted portfolio risk-based model skewness is at 99%.
Rational investors will invest in one of two ways:
In situations with a greater return on the investment risk position is the same.
In the same efficient investment position selects a location with less risk.

The result is not compatible with any of the two cases. In other words, the portfolio return is selected on the basis of skewness model is based on three levels of %14, %15 and %14As well as risk-based model skewness more than the classic version, and can be helpful in investment activity in the country.
REFERENCES