Analysis of waveguides coupling in Photonic Crystal Power-Splitter

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ABSTRACT: In this paper, we suggest a new power-splitter in two-dimensional photonic crystals which can be used for photonic integrated circuits. The proposed power-splitter mechanisms is similar to three-waveguide directional couplers and uses coupling between the states guided with Line defect waveguides. The input waveguide is divided into an image with two folds (two upper and lower waveguides). A photonic crystals power-splitter has been designed with FDTD method. Creation of coupling region in photonic crystal structure causes the wave to be coupled in this region and be transferred to the upper and lower waveguide and to have almost equal transmitted wave in two outputs A and B. In this way, the transmission rate can be reduced in input branch and wave can be equally transferred to upper and lower waveguides with almost equal transmission rate.

Keywords: photonic crystal, FDTD method, waveguide, coupling, power-splitter, point and linear defect, Temporal coupled mode theory.

INTRODUCTION

Light and matter have equal nature and this apparent duality results from our lingual limitations (Henry Heisenberg). With progress of technology, construction of small and smaller electronic parts has continued until it has been predicted that one can no longer construct the smaller parts which are able to conduct electricity flow in the next years. This discussion started when photonic crystal entered the science and then scientists in photonics branch were led to use photon instead of electron which is one of the discussed devices of photonic crystals. Any structure of which refractive index changes periodically is called photonic crystal. Photonic crystals are the structures which bright (electromagnetic) waves can pass in special frequency ranges. Photonic crystals have made of periodical dielectric metals and change electromagnetic waves radiation similarly to semi-conductive crystals which affect the presence of electrons due to periodicity of electric potential (Joannopoulos et al., 2008). Yablonovich first raised periodical structures from dielectric materials. These networks had high ability to control light (Yablonovich, 1987). To construct photonic crystal blades with GaInAsP semi-conductive element and air holes, drilling process was performed by the researchers, Baba and Inushita. This material which is a semi-conductive material is a useful material for being used in light emitting nanostructures because it has small recombination level. This work has been performed on point defect, line defect and combined defect cavities. In this paper, we introduce photonic crystal devices, cavity and waveguide, mention their properties, advantages and some of their applications in industry. Then, we analyze combined systems, Temporal coupled mode theory and present our analysis in this field. Considering type of application of the desired devices, any change which we make in crystal leads to creation of new devices which are based on photonic crystal principles. Here, Finite Difference Time domain (FDTD) has been used for simulation. FDTD method is a computational technique which is applied in different fields such as engineering, physics etc., studied and used for simulation of antennas, electronic circuits, nanoparticles, photonic crystals devices and etc. (Park et al., 2004). During the past years, several photonic crystals have been considered and several interesting characteristics have been shown as well. Many functional devices utilizing photonic crystals (PCs) have been proposed and are expected to play an important role in future optical circuit. The components being used in planar optical circuits include couplers, de-multiplexers/distributor, power-splitter, cavities, etc., and they can be identified in PC slab structures (Cross, 2003). The devices on the two-dimensional PC have several advantages such as relatively easy fabrication and convenient...
integration into conventional devices. Among of functional devices mentioned above, a power-splitter is one of the most important components. Anyway, power splitting function has been executed in photonic crystals and has a typical Y-junction inactive structure which has poor transmittance without structural tuning and conditions for zero reflection (Bosclo et al., 2002 and Johnson et al., 2001). In addition, although function of Y-junction can be improved with such tunings, Y branch which is composed of the tuned Y-junction and bending at output port, it has some problems which cannot be easily mentioned for practical applications which are rooted in incorrect adaptation of Y-junction mode and bending losses of output ports (Cross, 2003). For efficient power splitting in this paper, it is important to keep structure of devices symmetrically based on the input waveguide and finally, a three-waveguide structure with the central waveguide used as input port is produced (Park et al., 2004). With this subject, we designed a structure which shows the highest transmission rate up to 60% and average transmission rate up to 47% in each output waveguide in frequency of 0.258(\(a/\lambda\)).

**Photonic crystal devices**

Photonic crystals are the repetitive and regular structures of dielectrics with different refractive index. Photons can pass through photon crystals dependent on their wavelength or be reflected and call the wavelengths which are not permitted to pass as photonic band gap and are similar to electron band gap in semiconductors (Joannopoulos et al., 2008). A band gap can be created in the structure in different ways and this gap causes limitation in propagation of photon in one direction. These energy gaps cause to use these crystals for construction of mirrors, waveguides and microcavities with high efficiency and construct all-optical computer circuits. Today, these devices are used in displays, Light-propagating Diode, solar cells, optical sensors, diode lasers etc., in optical fiber and laser are considered based on structures of photonic crystal. These crystals provide opportunity for interaction of photon with material. If repetition of periodical structure is in one dimension, the formed crystal is called one-dimensional photonic crystals. Repetition of periodical structure in two and three dimensions will create two and three dimensional photonic crystals. In fact, this subject results from similarity of Schrödinger equation in solid state physics and Helmholtz equation in field. Refractive index plays the same role in Helmholtz equation as electric potential plays in Schrödinger equation. Therefore, function of photonic crystals (structures with periodical refractive index) against photons is similar to function of semi-conductive crystals (structures with periodical electric potential) against electrons. This behavior in these structures is similar to the behavior which metal and crystalline structures show against transmission of electrons. As It is accepted, electrons have wave behaviors and when they enter crystalline structure (regular and periodical) of atoms and are in special energy levels cannot transmit through the structure due to periodical structure of crystals and reflection of electrons. Now, if one can create an environment in which the photons with special energy cannot transmit through the structure, the control which was applied on behavior of electronics before was expected to be applied on behavior of photons. Photonic crystals are called crystals because their structure is similar to atomic crystals in terms of being regular and periodical and are called photonic because there are band gaps for photons (compared with electrons). To provide suitable opportunity for effective use of photonic crystals structure, there should be changes in periodical configuration of these crystals based on function of the desired devices. Each of the changes leads to creation of new devices based on principles of these crystals. In this paper, two examples of these devices i.e. filters and waveguides are studied. We study these structures in two dimensions. We consider an example of these structures i.e. dielectric bars with air environment. Point and line defects can be created in two ways. Basis of formation of photonic crystal resonance cavity is creation of point defect. This pointy defect can be created by changing dielectric constant of one or more bars or increasing or decreasing radius of its cross-section or combination of both compared with other dielectric bars. The use of line defect creates photonic crystal waveguide and their cross section is created by increasing or decreasing radius. Waveguides have been produced by creating line defects by omitting N row of pores in photonic crystal which is called \(W_n\) waveguide. Advantage of the waveguides is that they replace light in band gap plane (Yablonovich, 1987).

**Coupled mode theory**

One of the methods for study of photonic crystal is transient coupled mode theory. This theory describes the system based on ideal components (such as cavities and insulated waveguides) which have been disrupted or coupled. This method is formulated based on special expansion of modes of an ideal system and with damping rhythm and frequency. Then, it presents quantitative computations for the desired geometry. This method is based on five general principles of linearity, weak coupling, conservation of energy and invariance and the most important condition in this discussion is poor coupling (Joannopoulos et al., 2008).
For the above system, A is electric field domain in cavity and $\tau_1$ and $\tau_2$ are damping length of cavity to waveguide 1 and waveguide 2. $S_{1+}$ and $S_{2-}$ are input and output mode powers of waveguide 1 (Johnson et al., 2005). If we consider cavity resonance frequency as $\omega_0$, we will have:

\[
\begin{align*}
A(0) &\exp(-i\omega_0 t - \frac{t}{\tau_2}) + \frac{1}{\tau_1} = \frac{1}{\tau_2} A(t) = \frac{1}{\tau_2} \frac{\alpha_1 S_{1+} + \alpha_2 S_{2+}}{A(t)} \\
\end{align*}
\]

Relation 1 shows pure life of the system and Relation 2 shows variable field domain. Electromagnetic energy is $|A|^2$ and considering Poynting vector of $S=\frac{1}{2} \Re (E \times H)$, this quantity is also based on $|A|^2$ i.e. electromagnetic energy density. Considering Relations 1 and 2, the following relations can be written for the above system:

\[
\begin{align*}
\frac{dA}{dt} &= -i\omega_0 t - \frac{A}{\tau_2} + \frac{A}{\tau_1} + \alpha_1 S_{1+} + \alpha_2 S_{2+} \\
S_{1-} &= \beta_1 S_{1+} + \gamma_1 A \\
\beta & \text{ is reflection coefficient and } \alpha \text{ and } \gamma \text{ are coupling powers (Joannopoulos et al., 1997). By performing mathematical computations, the discussed general relations of the systems are obtained as follows:} \\
\frac{dA}{dt} &= -i\omega_0 t - \sqrt{\sum_{L=1}^{2} \tau_L} - \sqrt{\sum_{L=1}^{2} \tau_L} S_{1+} \\
S_{1-} &= -S_{1+} + \sqrt{\tau_1} A \\
to \text{ obtain transmission and reflection of this system, we assume that } S_{2+} = 0. \text{ It means that we don’t have input power to waveguide 2 (Joannopoulos et al., 1997). On this basis, reflection and transmission spectrum for the desired system is obtained as Relations 7 and 8.} \\
\frac{(\omega - \omega_0)^2 + \frac{1}{\tau_1^2}}{(\omega - \omega_0)^2 + \frac{1}{\tau_1^2} + \frac{1}{\tau_2^2}}\pi(\omega) R \\
(\nabla \times \left( \frac{1}{\epsilon(r)} \nabla \times H(r) \right) ) = \left( \frac{\omega}{c} \right)^2 H(r) \text{ (9)}
\end{align*}
\]

Relation 8 shows a Lorentzian peak with maximum in $\omega = \omega_0$ where $\omega_0$ is resonance frequency. Relations 7 and 8 can show conservation of energy relation, $R(\omega_0) + T(\omega_0) = 1$ (Joannopoulos et al., 2008).

Finite Difference Time domain (FDTD) to solve the equations, FDTD method has been selected which uses non-matrix process and recursive process for solving equations unlike other methods and there is no need for repetition of computation with one change. In this method, simulation space is divided into square cells and components of the fields are placed on each cell with special order.
Then, a set of relations is obtained for calculation of different components of field in terms of the related values at the previous times by applying finite difference on the Maxwell’s time dependent equation. For example, component X of electric convection vector is calculated from the following Relation:

$$D_{z} \mid _{i,j}^{n+1/2} = D_{z} \mid _{i,j}^{n-1/2} + \frac{\Delta t}{\Delta x} \left( -H_{y} \mid _{i,j}^{n+1/2} + H_{y} \mid _{i,j}^{n-1/2} \right)$$

$$+ \frac{\Delta t}{\Delta y} \left( H_{x} \mid _{i,j+1/2}^{n+1/2} - H_{x} \mid _{i,j-1/2}^{n+1/2} \right)$$

(10)

i and j specify location of cell, $\Delta x$ and $\Delta y$ are cell size, time index and $\Delta t$ is time step which is used in simulation. In this algorithm, the discussed fields are initialized with suitable primary conditions and then new values are obtained for each component in a time loop with the adjacent fields in a backward time step. After simulation time, the desired physical quantities can be calculated with values of fields in all cells (Taflove and Hagness, 2005 and Jalali and Mohammnadi, 2005).

**Simulation**

The structure includes an array of air holes with a triangular network structure. Air holes with a radius of $r=0.3a$ are perfected in GaAs($n=3$), where $a$ and $n$ are lattice constant and refractive index. In this structure, band fission opens for the frequency range of $0.2303-0.2666(a/\lambda)$ for $H$-polarization where $\lambda$ is the wavelength in free space.

The device designed here has the power-splitting mechanism like three-waveguide directional couplers can be divided into two regions based on their function i.e. coupling region and output region as shown in Figure 1. In coupling region, the emitted input field is coupled in the middle and on two sides of the waveguides and then the power of splitter is transferred to output waveguide A and B without coupling from one waveguide to another.

Figure 2. Networking of FDTD simulation space and location of electric and magnetic field components on each cell.

Figure 3. Geometry of photonic crystal power-splitter and setup for the Finite Difference Time domain (FDTD) computational. The above device can be divided into two regions according to their functions; Black boxed areas show photonic crystal waveguide. White holes represent air holes inside GaAs.

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waveguide in the designed frequency range. Air holes have radius of smaller than 0.2a among the waveguides to increase power of coupling in the coupling region.

\[
\psi(x, z) = C_0 \psi_0(x, z) e^{-j\beta_0 z} + C_2 \psi_2(x, z) e^{-j\beta_2 z}
\]

where \(C_m\) is field excitation coefficient and \(\psi_m(x, z) e^{-j\beta_m z}\) is localized Bloch wave function with propagation constant of \(\beta_m\). Index \(m\) denotes order of mode. After propagation coupling length, \(\psi(x, L)\) should satisfy the following condition so that the two-folded image can be created:

\[
\psi(x, L) = C_0 \psi_0(x, L) e^{-j\beta_0 L} + C_2 \psi_2(x, L) e^{-j\beta_2 L} = \frac{\pi}{|\beta_2 - \beta_0|}
\]

by evaluating Relation 12, the term for coupling length is obtained:

\[
L = \frac{\pi}{|\beta_2 - \beta_0|}
\]

propagation constants of \(\beta_0\) and \(\beta_2\) are obtained in \(a/\lambda=0.258\) with coupling length of \(L=19.762846\)a by substituting \(\beta_0 = 0.2682\) and \(\beta_2 = 0.02953\) in Relation 13. The coupling length is 20a, because the photonic crystals have discrete structure in terms of size. When coupling length of \(L\) is obtained, we confirm this problem by calculating Finite Difference Time domain (FDTD)(Park et al., 2004). We used the FDTD method for modeling propagation in coupling. In this method, we define the structure by networking computational space and allocating suitable optical properties to each cell. Longitudinal magnetic polarization(TM) has been considered. Then, we calculate the relations governing this structure with coupled mode theory. The computational domain is surrounded by the superposed layers for absorption of output waves. We considered the network as triangular. The computational space of \(65\times10\) is lattice constant. Simulation has been done for 50000 time intervals by PC. We presented result of transmission rate of two detectors through each waveguide in the following Diagram.
CONCLUSION

In this paper, FDTD method has been used to study the photonic crystal power-splitter with coupling waveguides for simulation and optimization. This structure has composed of three waveguides and coupling phenomenon occurs by crating defect in input and middle waveguides. The input wave is transferred to the upper and lower waveguide when it passes through the middle waveguide with coupling and we will have the equally transmitted wave rate in two outputs A and B. with this technique, wave can be transferred to the upper and lower waveguides almost equally and with 47% approximate transmission rate in each branch of power-splitter.

REFERENCES

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