Optimal Placement of PMUs Considering Sensitivity Constraints

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ABSTRACT: Phasor Measurement Unit (PMU), which is based on the GPS technique, is able to provide power engineers with immediate and precise measurements. By utilizing PMUs, the reliability and stability in power system are expected to be improved. This paper presents an optimization model to calculate the minimum number of PMUs for complete observability of electrical power networks. In addition to the goal minimizing the number of PMUs, the sensitivity indicator is developed and introduced into the PMU placement in order to enhance the security of the power system networks. Bus sensitivity analysis based on Decoupled Power Flow method is studied. Also, discussed will be about the effect of zero injection busses on the problem results. The Integer Linear Programming (ILP) approach is implemented to solve this problem. Numerical studies are conducted on the IEEE 14, 30, 57 and 118 busses systems and the results are analyzed.

Keywords: PMU placement, Integer Linear Programming, Sensitive bus, Sensitivity Indicator, observability.

INTRODUCTION

Phasor Measurement Unit (PMU) based on GPS technique is widely used to monitor the state of a power system. The PMU measurement is received by time sampling based on the same time reference synchronized by the GPS, so it could provide power engineers with immediate and precise measurements. By applying the PMU measurements in different areas in power systems such as state estimation, protection, load shedding, voltage collapses etc., the reliability and stability in power system are expected to be improved (Schweitzer and Whitehead, 2009). However, due to a high cost of PMUs or nonexistence of communication facilities in certain buses, it is impossible to place a PMU on every bus in the network, either as a stand-alone unit or relay-based function. So an optimal placement of PMUs is required for better power quality.

In order to make a good use of Phasor Measurement Units (PMUs), PMUs are recommended to be installed at certain buses that are critical and sensitive in the network. A sensitive bus refers to a bus which may have the maximum voltage phasor variation due to the system changes. On one hand, the system change could be the load increase.

In this paper, after the introduction of the sensitivity index and obtaining the list of most sensitive buses, the next step is to find an optimal PMU placement. In the previous PMU placement techniques, most methods only considered that an optimal PMU placement is to use the minimum number of PMUs to make the entire system observable but ignored the sensitivity analysis. Some of the previous papers showed the techniques of PMU placement considering bus sensitivity analysis, but two main drawbacks exist in these methods. One drawback is that the computational burdens of these methods are heavy; the other is that in these methods an optimal PMU placement without bus sensitivity analysis is required to be solved at first and then the final placement scheme considering sensitive buses is obtained by modifying the previous solved placement. Thus the process of minimizing the number of PMUs and locating some PMUs on the most sensitive buses are two separate processes which may reduce the range of the optimal solutions.

In reference (Flatabo et al., 1990), the authors derived the sensitivity indicators based on the nonlinear Jacobian matrix in load flow. The goal of this reference is to obtain the optimal reactive power reserve plan by reaching the mathematical formulations showing the relations between the voltages and the generated reactive power. This method works well for bus sensitivity analysis, however, the computation of this method is slow and complicated because the Jacobian matrix is nonlinear.

The author developed an index called VCPI (Voltage Collapse Prediction Index) based on the load flow equations in reference (Balamourougan et al., 2004). This method works well for one desired bus in each simulation by increasing the load on the corresponding bus and analyzing the bus voltage change compared with its neighboring buses. It only requires the system configuration parameters and the voltage phasor.
measurements at all buses in a computer program. This method has less computation burden and really works well for bus condition, however, it cannot offer a ranking of the most sensitive ones that is required in this research.

In reference (Nizam et al., 2007), the author reduced the system into a 2-bus equivalent system and then derived a sensitivity indicator showing the relation between the voltage changes and load increase based on this 2-bus system. This method is capable of giving the sensitivity rank. However, reducing a large and multi-generator system into a simple 2-bus system is complicated and the reactive power required in this indicator cannot be directly measured. Computing the P-V and Q-V curves at selected buses is another way of analyzing the voltage stability (Yorino et al., 1992; Taylor, 1994). In this method, the P-V and Q-V curves are only solved at one selected bus in each simulation. Each simulation contains a large number of runs of power flow by gradually increasing the load on a certain bus from its base value to its maximum value which may cause system blackout. After all P-V and Q-V curves have been derived, these curves are compared to attain the most sensitive buses. This method could display a detailed profile of bus sensitivity at each load level. However, for a large power system, this method is really time-consuming and it is not suggested to apply this method in real-time power system control.

In reference (Zhao et al., 2006), the author presented a new eigenvalue method. In this method, the author built a special matrix, for which the eigenvalues are associated with various operating parameters including the active injection power, reactive injection power and P-V nodes voltages. The relation of the P-V node voltage change and load increase could be further derived by eliminating the eigenvalue variable and only leaving the load injection and voltage variables in an equation. However, this approach should be solved in a nonlinear condition and has a heavy computational burden.

In this paper, the process of minimizing the number of PMUs and installing certain PMUs on the most sensitive buses are run together in a single program. The application of Integer Linear Programming (ILP) into the PMU placement problem are discussed in this paper.

BASIC MODEL IN PMU PLACEMENT

A PMU installed on a certain bus is able to measure the voltage magnitude and phase angle of the local bus and the branch current phasor of all branches emerging from this bus. The voltage magnitude and phase angle of the neighboring bus can be computed using voltage drop equations. Thus the buses monitored by a PMU are directly observable, the neighboring buses connected to the PMU buses are indirectly observable and the other buses which are not associated with the PMU buses are unobservable (Zhao, 2010).

Now the optimal placement of PMUs can be formulated as a problem of Integer Linear Programming (ILP) (Xu and Abur, 2004):

\[
\begin{align*}
& \text{Min} \sum_{i=1}^{n} w_i x_i \\
& \text{s.t.} \quad y = A_{n \times n} X_{n \times 1} \geq b_{n \times 1} 
\end{align*}
\]

Where \( n \) is total number of buses in the network and \( w \) is the cost function for the installed PMUs or the weight matrix for the buses that can vary based on the importance of every bus. \( w \) is normally equal to unit \( n \times n \) matrix. In this equation, \( x, A \) and \( b \) are defined as bellow:

\[
A_{n \times n}(i,j) = \begin{cases} 
1 & \text{if } i=j \\
1 & \text{if buses } i \text{ and } j \text{ are connected} \\
0 & \text{otherwise} 
\end{cases}
\]

\[
x_{n \times 1}(i) = \begin{cases} 
1 & \text{if PMU installed in bus } i \\
0 & \text{otherwise} 
\end{cases}
\]

\[
b_{n \times 1} = [1 \ 1 \ 1 \ldots \ 1]^T
\]

The inequality in function (1) is used for complete monitoring the system. The \( \ell \)th row in \( Ax \) matrix is the number of times that the \( \ell \)th bus is monitored which should be at least one.

Modeling Of Zero Injection Busses

For modeling of zero injection busses in the ILP framework a new variable that is called \( U_i \) is defined to confirm the monitoring ability for bus \( i \). If \( U_i = 1 \), it means that the \( i \) bus can be monitored and \( U_i = 0 \) means that \( i \) bus cannot be monitored. The set of busses that are connected to zero injection bus are called \( A_i \) and the \( A_i \)
with zero injection bus are called $B_i (B_i = A_i \cup \{i\})$. Any zero injections cause a new constraint that this condition is formulated in ILP as function (2) (Dua et al., 2008).

Objective : \( \min \sum_{i=1}^{n} w_i x_i \)

Subject to :
\[ Ax \geq u \]

and
\[ u_j = 1 \quad \forall j \not\in B_1 \cup B_2 \ldots \cup B_z \] \hspace{1cm} (2)

and
\[ \sum_{k \in B_i} u_k \geq |A_i| \quad \forall i \in Z \]

or
\[ a_i u \geq |A_i| \]

Here, $B_i = A_i \cup \{i\}$ and $|A_i|$ is the size of $A_i$ set.

**SENSITIVITY ANALYSIS**

In order to make a good use of Phasor Measurement Units, PMUs are recommended to be installed at certain busses that are critical and sensitive in the network. When a fault or a contingency occurs in a power system, the PMU is required to immediately and directly watch the power flow changes on the most severe transmission line. Through analyzing the measurements from the PMU at sensitive busses, power system operators should be capable of detecting the power system situation and taking the proper action to avoid any blackout quickly.

**Sensitive Transmission Line Analysis**

A sensitive transmission line refers to a branch which may have a larger amount of power flow rise resulting in overload due to the entire system load increase. A PMU is recommended to be located at either terminal of a sensitive transmission line in order to monitor the branch current phasor and further reflect the line loading condition.

The absolute value of complex power on line $i-j$ is equal to $|S_{ij}| = |V_i^* V_j^*|$. In a static power flow analysis, the voltage magnitudes are close to 1 at all busses or $V_i \approx 1$. So the partial derivative $\frac{\partial |S_{ij}|}{\partial P}$ reflecting the power flow changes with respect to system load increase is taken as the transmission line sensitivity indicator on line $i-j$, where $P$ is the summation of active loads in the entire system. Based on the approximation of voltages ($V_i \approx 1$), the sensitivity indicator is also equal to (Makram et al., 2011):

\[ \frac{\partial |S_{ij}|}{\partial P} = \frac{\partial |I_{ij}^*|}{\partial P} = \frac{\partial |I_{ij}|}{\partial P} \] \hspace{1cm} (3)

The current phasor through line $i-j$ could be expressed as:
\[ I_{ij} = (V_i \angle \delta_i - V_j \angle \delta_j) Y_{ij} \angle \theta_{ij} \] \hspace{1cm} (4)

Take the norms on both sides of this equation, it equals to:
\[ |I_{ij}| = |V_i \angle \delta_i - V_j \angle \delta_j| \] \hspace{1cm} (5)

According to the cosine law, the norm of the phasor difference between voltage phasor $i$ and $j$ could be expressed by a quadratic term:
\[ |I_{ij}| = \sqrt{V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_{ij})} \] \hspace{1cm} (6)

Differentiating the above equation with $\delta_{ij}$, the following equation could be concluded:
\[
\frac{\partial |I_{ij}|}{\partial \delta_{ij}} = \frac{V_i V_j \sin(\delta_{ij}) |Y_{ij}|}{\sqrt{V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_{ij})}}
\]  

(7)

According to the power flow equations, the active power at bus \(i\) is:

\[
P_i = V_i \sum_{j=1}^{n} V_j \left( G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij} \right)
\]  

(8)

The entire active power in a system is:

\[
P = \sum_{i=1}^{n} \sum_{j=1}^{n} V_i V_j \left( G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij} \right)
\]  

(9)

Arrange the equation above, it obtains:

\[
V_i V_j \left( G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij} \right) = P - \sum_{i'=1}^{n} V_i' \sum_{j'=1}^{n} V_j' \left( G_{i'j'} \cos \delta_{i'j'} + B_{i'j'} \sin \delta_{i'j'} \right)
\]  

(10)

Then,

\[
\delta_{ij} = -\tan^{-1}\left(\frac{G_{ij}}{B_{ij}}\right) + \sin^{-1}\left(\frac{1}{V_i V_j} \left( P - \sum_{i'=1}^{n} V_i' \sum_{j'=1}^{n} V_j' \left( G_{i'j'} \cos \delta_{i'j'} + B_{i'j'} \sin \delta_{i'j'} \right) \right) \right)
\]  

(11)

Differentiating \(\delta_{ij}\) with \(P\), it gives:

\[
\frac{\partial |I_{ij}|}{\partial P} = \frac{1}{V_i V_j \cos(\delta_{ij} + \beta)} \cdot \beta = \tan^{-1}\left(\frac{G_{ij}}{B_{ij}}\right)
\]  

(12)

Based on the partial derivatives deduced above, the transmission line sensitivity indicator is proved to be:

\[
\frac{\partial |I_{ij}|}{\partial \delta_{ij}} \frac{\partial \delta_{ij}}{\partial P} = \frac{1}{\cos(\delta_{ij} + \beta)} \cdot \frac{\sin(\delta_{ij}) |Y_{ij}|}{\sqrt{V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_{ij})}}
\]  

(13)

In order to show the influence on loading condition caused by the load increase, a modified sensitivity indicator divided by the power capacity on line \(ij\) is recommended. The indicator is:

\[
\frac{\partial |I_{ij}|}{\partial P} \frac{S_{i-j,\text{Lim}}}{S_{i-j,\text{Lim}} \cos(\delta_{ij} + \beta)} \cdot \frac{\sin(\delta_{ij}) |Y_{ij}|}{\sqrt{V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_{ij})}}
\]  

(14)

The sensitivity indicators at all busses are calculated and ranked. The terminals pertaining to the top sensitive lines are candidate PMU busses in priority.

**Pmu Placement Problem Formulation Considering Sensitivity Constraints**

In this case the placement problem could be formulated by the following equations:

\[
\min \sum_{i=1}^{n} w_i \cdot x_i
\]

Subject to:

\[
A_{n \times n} X_{n \times 1} \geq b_{n \times 1}
\]

\[
A'_{n \times n} X_{n \times 1} = b'_{n \times 1}
\]

\[
X_{n \times 1} = [x_1, x_2, \ldots, x_n]^T
\]

\[
x_i \in \{0, 1\}
\]

In equation (15), the definition of matrix \(A_{n \times n}\), vector \(b_{n \times 1}\), and vector \(X_{n \times 1}\) are the same as the basic model shown in equation (1). The equation \(A'_{n \times n} X_{n \times 1} = b'_{n \times 1}\) is the constraint that allocates the sensitive busses with corresponding PMUs. The matrix \(A'_{n \times n}\) and vector \(b'_{n \times 1}\) are defined as below:
\[ A_{i,j} = \begin{cases} 1 & \text{if } i=j \text{ and bus } i \text{ is a sensitive bus} \\ 0 & \text{otherwise} \end{cases} \]

\[ b_{i} = \begin{cases} 1 & \text{if bus } i \text{ is a sensitive bus} \\ 0 & \text{otherwise} \end{cases} \]

**Case Studies**

The proposed models are tested in IEEE 14, 30, 57 and 118 busses systems. The integer programming models in this study are solved using the function `bintprog` in MATLAB software package. Table 1 shows the sensitivity index for the transmission lines of IEEE 14 busses system that are calculated using equation (14). Similarly, the index can be obtained for the other systems studied.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Line</th>
<th>Sensitivity</th>
<th>Rank</th>
<th>Line</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4-5</td>
<td>0.1373</td>
<td>11</td>
<td>1-2</td>
<td>0.0452</td>
</tr>
<tr>
<td>2</td>
<td>7-9</td>
<td>0.1294</td>
<td>12</td>
<td>2-4</td>
<td>0.0430</td>
</tr>
<tr>
<td>3</td>
<td>9-10</td>
<td>0.1138</td>
<td>13</td>
<td>13-14</td>
<td>0.0412</td>
</tr>
<tr>
<td>4</td>
<td>6-13</td>
<td>0.0943</td>
<td>14</td>
<td>4-9</td>
<td>0.0397</td>
</tr>
<tr>
<td>5</td>
<td>10-11</td>
<td>0.0682</td>
<td>15</td>
<td>3-4</td>
<td>0.0335</td>
</tr>
<tr>
<td>6</td>
<td>6-11</td>
<td>0.0594</td>
<td>16</td>
<td>2-5</td>
<td>0.0300</td>
</tr>
<tr>
<td>7</td>
<td>4-7</td>
<td>0.0558</td>
<td>17</td>
<td>12-13</td>
<td>0.0298</td>
</tr>
<tr>
<td>8</td>
<td>6-12</td>
<td>0.0529</td>
<td>18</td>
<td>2-3</td>
<td>0.0272</td>
</tr>
<tr>
<td>9</td>
<td>5-6</td>
<td>0.0524</td>
<td>19</td>
<td>1-5</td>
<td>0.0238</td>
</tr>
<tr>
<td>10</td>
<td>9-14</td>
<td>0.0497</td>
<td>20</td>
<td>7-8</td>
<td>0.0005</td>
</tr>
</tbody>
</table>

Table 2 has the properties of the zero injection busses that are studied. The information related to these busses is obtained from reference (Power Systems Test Case Archive).

<table>
<thead>
<tr>
<th>Test system</th>
<th>No. of zero injection busses</th>
<th>Zero Injection Busses Locations</th>
</tr>
</thead>
<tbody>
<tr>
<td>14-bus IEEE</td>
<td>1</td>
<td>7,11,12,25,29</td>
</tr>
<tr>
<td>30-bus IEEE</td>
<td>5</td>
<td>6,9,11,21,22,24,26,34,36,37,39,40,45,46,48</td>
</tr>
<tr>
<td>57-bus IEEE</td>
<td>15</td>
<td>4,7,11,21,22,24,26,34,36,37,39,40,45,46,48</td>
</tr>
<tr>
<td>118-bus IEEE</td>
<td>10</td>
<td>5,9,30,37,38,63,64,67,71,81</td>
</tr>
</tbody>
</table>

Table 3 shows the placement of PMUs for the IEEE 14 busses system when the bus 5 and 9 are considered as sensitive busses in the network. Results are shown with and without modeling zero injection busses.

<table>
<thead>
<tr>
<th>Test System</th>
<th>Sensitive Busses</th>
<th>Status</th>
<th>No. of PMUs</th>
<th>Locations</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE-14</td>
<td>5, 9</td>
<td>Without Modeling Zero Injection Busses</td>
<td>5</td>
<td>4, 5, 6, 7, 9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>With Modeling Zero Injection Busses</td>
<td>4</td>
<td>2, 5, 6, 9</td>
</tr>
</tbody>
</table>

Tables 4 and 5 display the placement of PMUs for the IEEE 14, 30, 57 and 118 busses systems with and without modeling zero injection busses and also, considering sensitivity constraints. It should be noted that larger networks have more sensitive busses number that they can be directly monitored by PMU installation. Therefore, is considered two busses for 30-busses system, three busses for 57-busses system and four busses for 118-busses system as sensitive busses.

<table>
<thead>
<tr>
<th>Test System</th>
<th>Sensitive Busses</th>
<th>No. of PMUs</th>
<th>Locations</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE-30</td>
<td>6,11</td>
<td>10</td>
<td>1,2,6,10,11,12,19,24,25,30</td>
</tr>
<tr>
<td>IEEE-57</td>
<td>5,23,38</td>
<td>18</td>
<td>1,5,9,15,19,20,23,27,29,30,32,36,38,41,46,51,54,57</td>
</tr>
<tr>
<td>IEEE-118</td>
<td>4,34,56,116</td>
<td>33</td>
<td>3,4,7,9,12,15,17,21,25,28,34,37,42,45,49,52,56,62,64,</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>70,71,76,77,80,85,86,90,94,101,105,110,114,116</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test System</th>
<th>Sensitive Busses</th>
<th>No. of PMUs</th>
<th>Locations</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE-30</td>
<td>6,11</td>
<td>9</td>
<td>2,4,6,10,11,12,18,23,27</td>
</tr>
<tr>
<td>IEEE-57</td>
<td>5,23,38</td>
<td>14</td>
<td>1,5,9,19,23,25,28,32,37,38,46,50,53,56</td>
</tr>
<tr>
<td>IEEE-118</td>
<td>4,34,56,116</td>
<td>30</td>
<td>1,4,9,12,15,17,20,23,29,34,40,45,49,53,56,62,65,71,75,</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>77,80,85,86,90,94,101,105,110,114,116</td>
</tr>
</tbody>
</table>
Figure 1 shows the relationship between the required number of PMUs and the number of sensitive busses that should directly be monitored by the PMU for IEEE 118 busses system. Ten selected sensitive busses are obtained by introduced index in equation (14).

![Figure 1](image)

**Figure 1.** Number of PMUs required for complete observability the IEEE 118-busses system considering sensitivity constraints

As is shown in Figure 1, will increase the total number of PMUs requirements for complete observability of electrical power networks if increase the number sensitive busses need to be monitored directly by the PMU. It should be noted that with the constraints imposed ten sensitive busses to the placement problem, the number of PMUs has increased to number 3 that increase is not large compared with the number of sensitive busses. As a result, the system reliability can be increased up to considerable by study of sensitive busses in the network and PMU installed in them.

**CONCLUSIONS**

In this paper, a method in PMU placement considering zero injection busses in the ILP framework is proposed. This approach searches for the optimal solution in a global range and has a less computing burden. Furthermore the branch sensitivity indicator is developed and the PMU placement considering sensitivity analysis is studied. The simulation results show if more sensitive busses are required to be directly measured by PMUs, the total number of PMUs needed will be increased in order to make the entire system completely observable. However, the number of PMUs doesn’t significantly increase much. Actually in a real power system, extra PMUs are available sometimes so taking advantage of these extra PMUs is a challenging topic for power engineers.

**REFERENCES**


