Applying Information Gap theory mathematical model in optimizing portfolio

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ABSTRACT: Portfolio optimization is a discussion on forming a satisfactory portfolio which sometimes has faced with difficulties as there is an uncertainty in returning investment. Portfolio managers have usually intended to maximize portfolio efficiency for certain level of risk. As the current efficiency of assets is uncertain, then expected efficiency of portfolio which forms of assets can be depended upon considerable uncertainty. Information Gap Decision Theory has considered issue of building a robust portfolio against environmental uncertainty and provided a modern and creative approach for managing assets’ portfolio in term of sever uncertainty through applying a nonlinear mathematical model and somehow with intention of optimizing and gaining maximum profit (efficiency). In this research, mathematical model of the Information Gap Decision Theory has been applied in portfolio management for the first time in Iran. The model discusses a little in portfolio optimization by maximizing weights of assets and then the hypothesis has been proposed that efficiency of revised portfolio based on information gap theory is more than existed portfolio in the company which has been under study (Atieh Damavand Investment Company which selects items weights on the base of experience). According to analysis of results, it has been cleared that weights which were obtained by the model made portfolio efficiency of Atieh Damavand Investment Company nearly double than weights which were obtained in the company’s portfolio based on experience; it can confirm the proposed hypothesis.

Key words: Financial modeling, Information Gap Decision theory, Portfolio investment, Uncertainty management; Uncertainty model

INTRODUCTION

Financial issues has formed subject of many researches. Portfolio optimization is a vital activity in all organizations which has led to complicated and sometime conflicted processes of decision making.(Lin and Hsieh. 2004)

Various definitions have been provided for portfolio but one the most manifest of them has drawn from Wikipedia culture; on the base of Wikipedia encyclopedia (2009), portfolio is proper combination of assets which a person or company has kept.

Portfolio management means decision making on types and number of assets in portfolio with regard to portfolio owner's goals and economic conditions changes. Portfolio discussion is not only referring to select assets and securities; but portfolio also is a total balance for company which with different circumstances it has caused support of investor and provided chances for the person.

An investor must choose a cohesive portfolio in a way that has the most conformity with his/her needs.(Markowitz. 1959). Analysis of portfolio has been started with information relate to each investment institutions or securities and has been ended with total results of portfolios; therefore, purpose of portfolio's analysis is to find which portfolio can meet the investor's goals appropriately.(Markowitz. 1959)

Numerous studies have been conducted in the context of portfolio optimization which the majority of them are established with regard to the Markowitz approach and his proposed mathematical model which based on mean-variance and mainly followed purpose of minimizing portfolio variance for expected certain efficiency.(Markowitz. 1952)

However, it is cleared that all information relate to investment decision cannot be only base on capital return and risk but it can make better performance through considering more criteria as for the two factors; thus models have been gained importance which consider more criteria than Markowitz model.
It is obvious that there is a need to provide new optimization models as for inadequacy complicated mathematical models or imprecise concepts and variables and increasing growth in uncertainty and ambiguity sources and different approaches to control them. (Elton, Gruber et al. 1995)

**Information Gap Decision Theory**

Information gap decision theory is a quantitative and non-probabilistic method which was proposed for the first time by (Ben-Haim. 2006). The theory has been presented recently by (Sniedovich 2008) and along with the method Wald Maximum Model was suggested in part of classic decision theory (French January. 1986).

In (Ben-Haim. 2005) exercised various management models in relation to risk of financial markets which we have employed frameworks in area of information gap theory to create optimized and efficient portfolio in process of risk management. (Beresford-Smith. 2007)

One of the stimulants to use Info-Gap-Theory is being informed of probability limitations; problem of probability application is clear and demonstrable. In comparison with classic decision theory, information gap analysis has not applied probability distribution. In fact, deviation of error measures difference of a parameter and its estimation and does not measure result probability. Information gap is not sensitive to probable suppositions of results and has not utilized probability distribution. It can be claimed the theory is a tool for modeling and uncertainty management that the best application of the theory is for very complicated situation especially when there is strong deficiency in information.

The theory has played a role in modeling system and through this way help designer of model to select best model among different and incomplete information around that ideal model. The theory has contributed to decision making on the condition of uncertainty and followed the goal with using three models:

One of the models has created in condition which some parameters are uncertain (uncertainty model) and then next model has estimated under being error (robustness-consistency model) and the other is analyzing quality and quantity of outputs sensitively under error of estimation in this model and also it considers that if there is a close interdependence between process of creating an uncertainty model and decision making process (decision making model); the three model are describing as follows:

**Uncertainty Model**

The model starts with estimating parameters and has computed difference of other values of parameters from estimated values. If one of the uncertain parameters increases in estimation, set of possible values may be increased.

**Robustness-Consistency Model**

As for uncertainty model and minimum value of desired result for outputs and then for each decision, how uncertainty can guarantee obtaining to the minimum value of desired result? (Robustness of decision or robust of decision) In contrast, with desired output how uncertainty should make possible your access to ideal output? (Decision consistency) Uncertainty may be useful or harmful. The gap theory is based on quantity of two aspects of uncertainty and selecting one of them. Robustness function explains level of safety against failure while consistency function explains level of safety toward increasing wealth. The robustness function is laying ground of distinct decision making strategies and building a robust strategy of designing satisfaction (Feasibility of satisfaction maximizing under minimum conditions of survival).

**Decision Making Model**

Decision making is considered as optimization process on the base of robustness-consistency model with minimum output. When a decision is more robust, it can tolerate more uncertainty while it obtains desired result. (Robustness-satisfaction action) On the other hand, as for the desired result it is a decision which needs minimum uncertainty to access to output (Proper action). (Decision-theory-info-books)

**Research question**

With regard to environmental changeable conditions, can employing the method in managing portfolios of investment companies considered as a factor of risk reduction and the uncertainty? And has it finally caused portfolio more efficient than other methods?

**Research hypothesis**

Efficiency of revised portfolio based on information gap theory is higher than the current portfolio which weights of items have been selected on the base of experience.
METHODS

Information Gap Mathematical Model

We suppose that portfolio consists of N assets with weight of \( x_i \) and expected efficiency is \( r_i \), \( i = 1, \ldots, N \) which

\[
\sum_{i=1}^{N} x_i = 1
\]

\( x_i \) (weight of each asset in portfolio) has shown that amount of investment in each asset toward total investment in assets.

Maximizing weight of assets

In order to correct management and to control portfolio performance; it is appropriate idea to create a portfolio which is stable and efficient on the condition of uncertainty. Information gap theory has performed the purpose through maximizing weight of assets in portfolio.

Maximum weight (\( x \)) can be computed simply through optimizing the Lagrange Standard as follows:

\[
(3-1) \quad x_i^* = \frac{[c^{-1}z_i]}{\sum_{i=1}^{N}[c^{-1}z_i]}
\]

With considering limitation:

\[
\sum_{i=1}^{N} x_i = 1
\]

In this formula, \( c^{-1} \) is an invertible matrix of covariance matrix and \( z \) is a vector that can be computed through the following way:

\[
(3-2) \quad z_i = \tilde{r}_i - p
\]

In above formula, \( \tilde{r}_i \) is estimation return of share and \( p \) is model parameter which has been calculated through the following formula:

\[
(3-3) \quad p \leq \frac{\Sigma_{i=1}^{N} c_{ij}^{-1} \tilde{r}_j}{\Sigma_{j=1}^{N} c_{jj}^{-1}} \equiv b
\]

When maximum weight of portfolio’s items have been computed, return of new portfolio which is called stable optimized portfolio would be computed and compared with primary portfolio return (before applying the method). (ELTON, GRUBER et al. 1995)

Maximizing robustness

In this method maximum robustness of portfolio is computed through the following mathematical formula:

\[
(3-4) \quad \tilde{\alpha}(x^*) = \sqrt{z^T c^{-1} z}
\]

In the above formula, \( z \) is a vector with \( N \) sides consists of \( Z \) which can be obtained by the formula (3-2), \( Z^T \) is transpose of vector \( Z \), \( C^{-1} \) is an invertible matrix of covariance matrix (\( C \)) and \( \tilde{\alpha}(x^*) \) is maximum robustness of optimized portfolio. All limitations of created efficiency would be true for \( \tilde{r} \).

\[
(3-5) \quad 0 \leq x^T \tilde{r} - p = \frac{z^T c^{-1} z}{\Sigma_{i=1}^{N}[c^{-1}z_i]}
\]

As for the value of \( C \) is determined and positive based on supposition, numerator of the fraction is positive in formula (3-4), so it is necessary to have a positive numerator of fraction in equation (3-4) in order to obtain maximize result in equation (3-2). The below relation is drawn from the formula (3-4):

\[
(3-6) \quad p \leq \frac{\Sigma_{i=1}^{N} c_{ii}^{-1} \tilde{r}_i}{\Sigma_{i=1}^{N} c_{ii}^{-1}} \equiv b
\]

As for the value of \( C \) is determined and positive, numerator of the fraction is positive in formula (3-6); but denominator of the fraction can have any sign. (Elton, Gruber et al. 1995)
During operation managers have always looked for positive portfolios and then it is necessary to have \( P > 0 \).

When the value of \( b \) is less than \( P (b<P) \), value of \( x_i^* \) can be defined by equations (3-1) and (3-2), so the value of unstable return is not maximum. In this example lines of efficiency and robustness must be considered carefully.

It is considerable that to select different weights of portfolio \((x=x_i)\), derivative of (3-1) and (3-2) as per certain values of \( p = \rho < b \) are lines which it is necessary the lines as a function of \( p \) has been tangent with \( \alpha(x^*, b) \) curve and had positive robustness for \( p > b \) which is an upper bound for \( \hat{\alpha} (x^*, b) \).

When the value of \( b \) is more than \( P (b>P) \), value of \( x_i^* \) can be defined by equations (3-1) and (3-2), the maximum robustness has been obtained. Its diagram is acquired through a straight line that means the value of robustness has not changed by increasing the value of \( x \).

Also to obtain different weights \( x_i^* = x_i \), derivative that has been computed through formulas (3-1) and (3-2) for certain values of \( p = \rho < b \) is defined with lines which are tangent of convex function \( \hat{\alpha} (x^*, b) \) that has positive robustness for \( p > b \) and is bounded from upper side by \( \hat{\alpha} (x^*, b) \).

Therefore, \( p = b \) is clear upper limit for expected profit (efficiency) unless some limitation add to weights (Maller and Turkington 2002).

**Stable optimal frontier**

\[ \hat{\alpha}(x^*, \rho) \]

![Diagram of robust optimal frontier](image)

Efficiency lines- robustness creates a long convex curve that covers line of robustness curve and the line has been called ROF- Robust Optimal Frontier in similarity with efficient frontier in CAPM- Capital Asset Pricing Model.

ROF curve as a function of \( p \) has been obtained through the following formula:

\[ (3-7) \quad \hat{\alpha}(x^*, \rho) = \sqrt{(\bar{\rho} - \rho)^2 c^{-1}(\bar{\rho} - \rho)} \]

Equation (3-7) can be simplified as follows:

\[ (3-8) \quad \hat{\alpha}(x^*, \rho) = a^2 f(p) \quad a = \sum_{i=1}^{N} c_i^{-1} \]

In this equation

\[ f(p) = \sqrt{\bar{\rho}^2 - 2bp + c} \]

\[ b = \sum_{i=1}^{N} c_i^{-1} \]

\[ c = \sum_{i=1}^{N} \left( \frac{c_i^{-1} f_i}{a} \right) \]

As for matrix \( c \) is certain and positive, the values of \( c \) and \( a \) are positive and also function of \( f(p) \) would be positive as per all values of \( p \); it is emphasized the quadratic equation of (3-9), expect \((b^2 - c > 0)\), is negative.

\[ (3-10) \quad f'(p) = \frac{b - b}{f(p)} \quad f'(b) = (c - b^2)(p^2 - 2p + c)^{-3/2} \]
As for the value of \((b^2 - c)\) is negative, the value of \(f'(p)\) will be positive (concavity of ROF curve is upward) and curve has reduced constantly in range of \(0 \leq p \leq b\) and derivative is obtained zero in \(p = b\). With regard to the above descriptions, in formula (3-7), when the value of \(P\) is less than \(b\) (\(P < b\)), robustness is optimized; and when value of \(P\) is more than \(b\) (\(P > b\)), we use structure of convex curve to obtain diagram ROF based on figure (3-1).

Therefore function of ROF has been defined as follows:

\[
\phi_{\text{ROF}}(x^*, p) = \begin{cases} 
  a^{1/2}f(p) & p < b \\
  a^{1/2}f(p) & p \geq b 
\end{cases}
\]

As the figure has shown, \(f'(p)\) is tangent with the curve at point \(p = b\) (its derivative is zero at \(p = b\)) and as the value of \(f'(p)\) will be positive, concavity of ROF curve is upward and curve has reduced constantly in range of \(0 \leq p \leq b\) and finally it becomes to a straight line as per values of \((p \geq b)\), it means that \(p = b\) is clear upper limit for profit (efficiency). On the other hand robustness has not changed when value of \(p\) is added (Beresford-Smith 2009).

**METHOD OF ANALYSIS**

In this study we have employed mathematical model of information gap theory to obtain maximum weights of assets in portfolio. EXCEL and MATLAB software have been used in the study.

**Organizing input data**

Efficiency and weight (portion of each item in Investment Company's portfolio) are required variables to implement the mathematical model of the study which data has been collected from Atieh Damavand Investment Company which is an investment company depended to stock exchange.

It should be mentioned that RAHAVARD NOVIN software which is specific software for the stock exchange organization has been employed to collect data in the study.

Table 1 has indicated estimation return, weight and assets return in portfolio which have been collected through RAHAVARD NOVIN software with information related to Atieh Damavand Investment Company.

<table>
<thead>
<tr>
<th>Row</th>
<th>Assets items</th>
<th>(R_i)</th>
<th>(R_{\text{f}})</th>
<th>(X_i)</th>
<th>(e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Corporate bonds and bank deposits</td>
<td>0.198864</td>
<td>0.44</td>
<td>0.017</td>
<td>0.241136</td>
</tr>
<tr>
<td>2</td>
<td>Oil products</td>
<td>0.06878</td>
<td>0.38</td>
<td>0.03</td>
<td>0.31122</td>
</tr>
<tr>
<td>3</td>
<td>Chemical material and products</td>
<td>0.010748</td>
<td>0.39</td>
<td>0.33</td>
<td>0.379252</td>
</tr>
<tr>
<td>4</td>
<td>Rubber and plastic</td>
<td>0.01E-02</td>
<td>0.43</td>
<td>1.10E-01</td>
<td>0.3499</td>
</tr>
<tr>
<td>5</td>
<td>Other non metal mineral products</td>
<td>0.008162</td>
<td>0.05</td>
<td>0.088</td>
<td>0.041838</td>
</tr>
<tr>
<td>6</td>
<td>Produce metal products</td>
<td>0.017051</td>
<td>0.093</td>
<td>0.04</td>
<td>0.085949</td>
</tr>
<tr>
<td>7</td>
<td>Radio, TV and communication means</td>
<td>0.099924</td>
<td>0.51</td>
<td>0.004</td>
<td>0.410076</td>
</tr>
<tr>
<td>8</td>
<td>Medical and optical measuring devices</td>
<td>0.017067</td>
<td>0.18</td>
<td>0.0241</td>
<td>0.162933</td>
</tr>
<tr>
<td>9</td>
<td>Industrial multi business companies</td>
<td>0.116834</td>
<td>0.88</td>
<td>0.047</td>
<td>0.308198</td>
</tr>
<tr>
<td>10</td>
<td>Financial mediatory</td>
<td>0.071802</td>
<td>0.38</td>
<td>0.15</td>
<td>0.308198</td>
</tr>
<tr>
<td>11</td>
<td>Computer and its related activities</td>
<td>0.21557</td>
<td>0.1</td>
<td>0.04</td>
<td>-0.11557</td>
</tr>
<tr>
<td>12</td>
<td>Technical and engineering services</td>
<td>0.010365</td>
<td>0.12</td>
<td>0.012</td>
<td>0.109635</td>
</tr>
<tr>
<td>13</td>
<td>Post and telecommunication</td>
<td>0.058877</td>
<td>0.2</td>
<td>0.0397</td>
<td>0.141123</td>
</tr>
</tbody>
</table>

With data of table 1, covariance matrix (table 2) and invertible matrix (table 3) have been computed by MATLAB software:
As it is cleared, because portfolio of Atieh Damavand Company consists of 13 assets, the covariance matrix a 13×13 matrix which each component has been obtained with multiplying weights in covariance between two assets in raw and column of that component and items which have placed on main diameter are variance of assets.

<table>
<thead>
<tr>
<th></th>
<th>X₁</th>
<th>X₂</th>
<th>X₃</th>
<th>X₄</th>
<th>X₅</th>
<th>X₆</th>
<th>X₇</th>
<th>X₈</th>
<th>X₉</th>
<th>X₁₀</th>
<th>X₁₁</th>
<th>X₁₂</th>
<th>X₁₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>X₃</td>
<td>-5.92E+19</td>
<td>-9.54E+19</td>
<td>-7.36E+18</td>
<td>4.92E+16</td>
<td>3.72E+20</td>
<td>1.90E+20</td>
<td>-4.02E+20</td>
<td>1.82E+20</td>
<td>-1.34E+19</td>
<td>-8.08E+18</td>
<td>1.06E+19</td>
<td>6.09E+20</td>
<td>1.28E+19</td>
</tr>
<tr>
<td>X₄</td>
<td>1.10E+18</td>
<td>1.72E+18</td>
<td>4.92E+16</td>
<td>4.96E+14</td>
<td>-6.75E+18</td>
<td>-3.11E+18</td>
<td>7.66E+18</td>
<td>2.89E+18</td>
<td>-1.16E+17</td>
<td>7.62E+16</td>
<td>-1.86E+17</td>
<td>-1.07E+19</td>
<td>-2.39E+17</td>
</tr>
<tr>
<td>X₁₁</td>
<td>-1.37E+19</td>
<td>-1.50E+19</td>
<td>1.06E+19</td>
<td>-1.86E+17</td>
<td>6.34E+19</td>
<td>-1.54E+19</td>
<td>-7.64E+19</td>
<td>-2.76E+19</td>
<td>2.25E+18</td>
<td>-9.89E+17</td>
<td>8.83E+17</td>
<td>5.62E+19</td>
<td>3.03E+18</td>
</tr>
</tbody>
</table>
Table 4. The outputs of \( p, Z \), and \( x^* \)

<table>
<thead>
<tr>
<th>( R_i )</th>
<th>( p )</th>
<th>( Z )</th>
<th>( x^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.28085</td>
<td>0.15915</td>
<td>0.040325</td>
<td></td>
</tr>
<tr>
<td>0.28085</td>
<td>0.09915</td>
<td>0.034429</td>
<td></td>
</tr>
<tr>
<td>0.28085</td>
<td>0.10915</td>
<td>0.002193</td>
<td></td>
</tr>
<tr>
<td>0.28085</td>
<td>0.14915</td>
<td>1.14E-05</td>
<td></td>
</tr>
<tr>
<td>0.28085</td>
<td>-0.23085</td>
<td>0.324607</td>
<td></td>
</tr>
<tr>
<td>0.28085</td>
<td>-0.19785</td>
<td>0.358028</td>
<td></td>
</tr>
<tr>
<td>0.28085</td>
<td>0.22915</td>
<td>0.37466</td>
<td></td>
</tr>
<tr>
<td>0.28085</td>
<td>-0.10085</td>
<td>0.128634</td>
<td></td>
</tr>
<tr>
<td>0.28085</td>
<td>0.59915</td>
<td>0.054103</td>
<td></td>
</tr>
<tr>
<td>0.28085</td>
<td>0.09915</td>
<td>0.001758</td>
<td></td>
</tr>
<tr>
<td>0.28085</td>
<td>-0.18085</td>
<td>0.004311</td>
<td></td>
</tr>
<tr>
<td>0.28085</td>
<td>-0.16085</td>
<td>0.245877</td>
<td></td>
</tr>
<tr>
<td>0.28085</td>
<td>-0.08085</td>
<td>0.004469</td>
<td></td>
</tr>
</tbody>
</table>

RESULTS AND DISCUSSION

Finally, efficiency of new portfolio and maximum value of robustness have been computed:

<table>
<thead>
<tr>
<th>Portfolio return with weights of ( x_i )</th>
<th>0.0392</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portfolio return with weights of ( x^*_i )</td>
<td>0.0657</td>
</tr>
</tbody>
</table>

One of the factors to measure portfolio profit (efficiency) is its return, portfolio efficiency of Atieh Damavand Investment Company has computed with primary weights and maximized weights (through the model implementation) and then registered in table 5. Maximum robustness for function of robustness = \( \delta (x^*) = 3.88 \times 10^{10} \)

In this study, one of the main decisions on financial and management issues, portfolio optimization, has been considered in field of modeling. The consideration was impossible in the past because of complexity of many mathematical models. In the present study, the possibilities have been tested and employed with regard to new approaches and developing abilities to solve mathematical advanced algorithms by computer.

Portfolio modeling in an uncertain and vague environment has discussed in the research. In fact we consider managing portfolio assets with uncertain efficiency through a non-probabilistic and non-linear model, as well as include mathematical complicated steps. So in order to response the research question, the hypothesis has been proposed in which "Efficiency of revised portfolio based on information gap theory is higher than the current portfolio which weights of items have been selected on the base of experience".

Portfolio efficiency from 0.392 has been reached to 0.657 by using the model and method of maximizing Lagrange which was applied for computing optimal weight of assets. It means that when the method was employed the portfolio efficiency has become double which proves the effectiveness of information gap theory model; it can confirm the proposed hypothesis.

As acceptable results were obtained on the conditions of uncertainty through using the model, therefore, portfolio’s managers have been advised to apply the model to optimize weights of different assets in portfolio and increase efficiency.

Research limitations

The study has analyzed only an uncertainty model in special time framework and has not considered assets efficiency of zero and negative.

REFERENCES


