Calculation of Strong Coupling Constant with Momentum Spectra and Renormalization Group Equation (RGE)

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ABSTRACT: Hadron spectra and parton spectra are proportional in condition of providing cut-off parton falling \( Q_0 \) reduce to the side of hadron mass. Considering to the production of hadron in low level energy has done in boundary of perturbative QCD and non-perturbative QCD, the study was conducted to calculate momentum spectra which is produced spectra in annihilation of electron-positron using AMY data in center of mass energy \( \sqrt{S} = 58 \text{ Gev} \). Then momentum distribution accompany with its precedent compared with predict Modified Leaning logarithmic Approximation and \( \Lambda_{\text{eff}} \) is calculated. Finally, strong coupling constant, \( \alpha_s \), will be calculated using \( \Lambda_{\text{eff}} \) and renormalization group equations. The results show \( \alpha_s \) in error limits, is consistent with amounts of reported in other energy levels.

Keywords: Effective scale factor, Momentum spectra, Renormalization group equation, Strong coupling constant, \( \beta(\alpha_s) \) function

INTRODUCTION

The theory of Quantum Chromodynamics is a theory accepted in describing interactions between quarks and gluons (Adler et al., 2003). This theory described the strong interaction among quarks regarding the interchanging of gluons (Sterman et al., 1995). In this theory strong coupling constant, \( \alpha_s \), depends on energy, which is diverged for long-orders (low level energy), and converged for short-orders (high level energy). Therefore, we can use perturbate calculation of measuring \( \alpha_s \) in high level energies. But in fundamentals lab and experiments, this theory is not usually useable and we should use phenomenology rudiment for better description. For this aim, annihilation of electron-positron is a forthcoming means to do research in QCD theory. This annihilation provides an appropriate way to calculate strong coupling constant. Regarding to the above matter, the data which is used in this paper provided from AMY detector, TRISTAN accelerator from Japan which prevents the collision between \( e^- e^- \) in center of mass energy 58 Gev, (Lee et al., AMY Collaboration, Phys. Lett, (1993)). In this paper, the data was analyzed and calculated their momentum distribution and finally, using distribution of momentum spectra and RGE-Renormalization Group Equation, \( \alpha_s \) will be calculated.

PHYSICAL RESULTS

The hadron momentum spectra is defined as following; (V.A. Khoze and W. Ochs, 1977):

\[
\xi = -\ln \left( \frac{2p}{\sqrt{S}} \right) \tag{1}
\]

\( P \) and \( \sqrt{S} \) are momentum spectra and center of mass energy's particles, respectively.

Formula 1 based on the formulation of the so called MLLA-modified leading-log approximation, (V. A. Khoze and W. Ochs, 1997). This approximation constitutes a compete re summation of single and double logarithmic terms (Dokshitzer et al., 1982). In the MLLA approach, a so called limiting spectrum prediction for the \( \xi \) distribution can be formulated in which the QCD scale, \( \Lambda \), and the scale at which the perturbative process terminates, \( Q_0 \), are taken to have the same value. In this study we use the equations given in (R. Perez-Ramos, F. Arleo & B. Machet, 2008):

\[
\frac{1}{\sigma_{\text{had}}} \frac{d\sigma}{d\xi} = 2K_{\text{LPHD}} \times f_{\text{MLLA}}(\xi, \Lambda_{\text{eff}}, N_c, n_f) \tag{2}
\]
$K_{LPHD}$ is normalization factor that describes the process of becoming hadron. $F_{MLLA}$ is complex function of $\xi$ and $\Lambda_{eff}$ is effective scale factor. $N_c$ and $n_f$ are color factor and the number of active quarks respectively (BES collaboration, arXiv:hep-ex/0306055 v2 1Jul 2003).

Figure 1 shows $\xi$ related to multiplicity of AMY data at 58 Gev center of mass energy. According to this figure, multiplicity is almost equal zero at $\xi=0$. By increasing $\xi$ up to 3.3, multiplicity will be increased and then decreased while $\xi$ approach to 5. Comparison this graph with the results of other data from distribution of momentum spectra in low level energy, (BES Collaboration, arXiv:hep-ex/0306055 v2 1 Jul 2003), reveals that by increasing the center of mass energy, $\xi$ will be wider and the peak values move to the bigger (higher) values of $\xi$. This result is confirmed by the other data at different center of mass energies, (Khoze and Ochs, Int J Mod Phys A12 2949 (1997), (Fig 3).

![Figure 1. calculated $\xi$ spectra in 58Gev for AMY data](image)

As it is shown, according to the distribution of momentum spectra in different energy level, (Figure 3 and particularly the 58Gev center of mass energy), all graphs are peaked shape. This peak is important and the value of $\xi$ can be calculated for each peak ($\xi^*$) and then the value of $\Lambda_{eff}$ can be calculated. MLLA shows dependence of the energy peak place ($\xi^*$) as follows (Khoze and Ochs, 1997):

$$\xi^* = \frac{1}{2} Y + \sqrt{cY} - c$$ \hspace{1cm} (3)

In the formula (3), $Y = \ln \left( \frac{0.5\sqrt{5}}{\Lambda_{eff}} \right)$, and $C$ is $0.2915, (0.3190)$, for four (five), active flavors. In order to calculate $\Lambda_{eff}$, first the graph of Figure 1 was calculated, by Gaussian function:

$$y = y_0 + \frac{A}{W} \exp \left( -2 \frac{(x-x_c)^2}{W^2} \right)$$ \hspace{1cm} (4)

The peak place of this graph is calculated by putting $y'=0$ in $x=x_c$. $x_c$ is the same as $\xi^*$. The coefficients of $y_0$, $A$, $W$ and $x_c$ are calculated by MRQMIN computer program which is used to calculate the above function. The results are as follows, (table 1 and Figure 2):

<table>
<thead>
<tr>
<th>$\xi = x_c$</th>
<th>$W$</th>
<th>$A$</th>
<th>$y_0$</th>
<th>$y^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.252</td>
<td>2.625</td>
<td>10.597</td>
<td>-0.369</td>
<td>1.379</td>
</tr>
</tbody>
</table>

Table 1. Calculated Coefficients of Gaussian Function
In Figure 3, calculated $\xi$ spectra which are obtained from AMY data, are shown with measurements of other tests (Brook and Skilicorn, 2000).

Substituting the result of $(\xi^*)$ in formula 3 to calculate $Y$:

$$Y=4.7367 ; \ C=0.2915 ; \ (four\ flavor)$$

$$Y=4.6941 ; \ C=0.3190 ; \ (five\ flavor)$$

By inserting the value of $\sqrt{S}$ in formula $Y = \ln \left( \frac{0.5\sqrt{S}}{\Lambda_{\text{eff}}} \right)$, the amounts of $\Lambda_{\text{eff}}$ are calculated:

$$\Lambda_{\text{eff}} = 263 \pm 8\ MeV ; \ C=0.2915 ; \ (four\ flavor)$$

$$\Lambda_{\text{eff}} = 274 \pm 6\ MeV ; \ C=0.3190 ; \ (five\ flavor)$$

The result of calculated $\Lambda_{\text{eff}}$ using AMY data, is consistent with the results of OPAL ($\Lambda_{\text{eff}}=263\pm4\ MeV$) (OPAL Coll., Ackerstaff et. al., 1999), ZEUS ($\Lambda_{\text{eff}}=251\pm14\ MeV$), (Jamieson, PhD thesis, DESY F35 D-95-01), and CDF.
\( \Lambda_{\text{eff}} = 256 \pm 13 \text{MeV} \) \((\Lambda_{\text{eff}}=256\pm 13 \text{MeV})\) (A. Safonov Talk at 8\textsuperscript{th} Int. Workshop on Deep Inelastic Scattering and QCD (DIS), 2000) at the error limits. Figure 4 shows \( \xi^* \) for different level of \( E_{\text{c.m}} \) and AMY data:

\[
\begin{align*}
\xi^* & \quad \text{spectra in different energies} \quad \sqrt{s} \quad \text{(GeV)} \\
\end{align*}
\]

The results show, \( \xi^* \) for AMY data is consistent with the other data in lower and higher level of center of mass energies, (TASSO Coll., W. Braunschweig, et al, 1990), (MARK Coll., A. Peterson et al, 1988), (TOPAZ Coll; Itoh et al, 1995), (ALEPH Coll; Abrev et al, 1995). The results are compared to the predictions of PYTHIA 6.2, HERWIG 5.8 and ARIADNE 4.11 and to the fitted prediction of equation (2). The fitted MLLA prediction is in good agreement with the data. The PYTHIA and ARIADNE predictions are slightly below the data, while the HERWIG predictions, in particular at the highest c.m energies is much lower than the data.

**Calculating the strong coupling constant**

Strong coupling constant of a special process depends on the scale of renormalization \( \mu^2 \). This running coupling constant can be calculated easily based on \( \beta(\alpha_s) \) function in renormalization group equation (Gell-Mann and Low, 1954).

\[
\mu \frac{\partial \alpha_s}{\partial \mu} = \beta(\alpha_s) = -\beta_0 \alpha_s^2 - \beta_1 \alpha_s^3 - \beta_2 \alpha_s^4 - \text{HigherTerms} \quad (5)
\]

Which we have:

\[
\begin{align*}
\beta_0 &= 11 - \frac{2}{3} N_f \\
\beta_1 &= 51 - \frac{19}{3} N_f \\
\beta_2 &= 2857 - \frac{5033}{9} N_f + \frac{325}{27} N_f^2
\end{align*}
\]

In this equation, \( \mu \) is a factor with mass dimension and introduced for converting of strong coupling constant to dimensionless (Bardeen et al., 1978). \( N_f \) is the number of quarks with different flavors. Strong coupling constant, \( \alpha_s \), is calculated as a function of \( \mu \) with integration of equation 5. Integration constant of \( \Lambda_{\text{eff}} \) is introduced to solve this differential equation. As mentioned above, \( \Lambda_{\text{eff}} \) is basic constant of QCD which calculated by trial and momentum distribution. After integration of formula 5, and considering the first three terms in the right side of the equation, finally running coupling in NNLO-Next to Next Leading Order, will be as follows:
\[
\alpha_s(\mu^2) = \frac{4\pi}{\beta_0 \ln\left(\frac{\mu^2}{\Lambda^2}\right)} \times \left[ 1 - \frac{2\beta_1}{\beta_0^2} \ln\left(\frac{\ln(\mu^2)}{\Lambda^2}\right) + \frac{4\beta_1^2}{\beta_0} \ln\left(\frac{\mu^2}{\Lambda^2}\right) - \frac{1}{2} + \frac{\beta_2 \beta_0}{8\beta_1^2} - \frac{5}{4} \right]
\]

(6)

Considering calculated \( \Lambda_{\text{eff}} \) in the 58Gev center of mass energy and bearing the formula 6, strong coupling constant equals \( \alpha_s = 0.115 \pm 0.008 \).

CONCLUSION

In this paper the data of 58 Gev center of mass energy was analyzed and \( \Lambda_{\text{eff}} \) was calculated which is boundary of perturbative QCD and non-perturbative QCD regarding momentum distribution. Momentum distribution was calculated by Gaussian function. In the error limits, \( \Lambda_{\text{eff}} \) of AMY data is consistent with other energies. Finally, using the \( \beta(\alpha_s) \) function in Renormalization Group Equation, strong coupling constant \( (\alpha_s) \) was calculated which equals to \( \alpha_s = 0.115 \pm 0.008 \).

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