Enhanced Geometry-Based MIMO Propagation Channel Modeling for 4G Networks

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ABSTRACT: Enhanced geometry-based Multiple-Input Multiple-Output (MIMO) channel models for wideband propagation channels of 4G networks are proposed. The contribution of the paper is three-fold. First, joint single- and double-bounce propagations are modeled by considering the scatterers located indoor on a hollow-disk around the user antennas. Second, the propagation channel model is derived when indoor scatterers are located on a hollow-disk around the user and a group of outdoor scatterers exist between the base station and the user. The outdoor scatterers cause double-bounce propagations and are located either as a compact form or distributed one. Third, the closed-form Cross-Correlation Functions (CCFs) are derived for the aforementioned scenarios. These functions depend on various parameters of interest including the distance between the base station and the user, the spacing among antennas elements, and the directions of the antennas elements. A numerical study is performed to calibrate/verify the CCFs. Finally, the capacity of the channel is derived. We compare the derived capacity with the one achieved by the Radiowave Propagation Simulator (RPS) package. The results show that considering the outdoor scatterers increases the accuracy of the capacity computation significantly.

Keywords: geometry-based MIMO channels, joint single- and double-bounce reflections, scatterers, cross-correlation function, capacity.

INTRODUCTION

Modern telecommunication networks, such as, Fourth Generation (4G) networks, are growing toward providing high capacity or data rate. Indeed, capacity improvement is achieved by deploying Multiple-Input Multiple-Output (MIMO) technology in 4G networks. MIMO gain, i.e., improved data rate with respect to the one's of a Single-Input Single-Output (SISO) system, is achieved by implementing spatial-multiplexing and transmission diversity techniques (Irtaza et al., 2012) on a multi-path propagation channel. Hence, to investigate the capacity performance of 4G networks deploying MIMO, we need to fully investigate the characteristics of multi-path propagation channels. Since channel characterization by measurement is complex, time consuming, and costly, a suitable alternative is channel modeling (Poutanen et al., 2011).

Many MIMO propagation channel models have been proposed in the literature (Almers et al., 2007); among them, the Geometry-based Stochastic Channel Models (GSCM) consider the statistical parameters of the environment and model the spatial structure of the channel in a convenient way with low complexity (Czink et al., 2007). By applying the fundamental laws of reflection, diffraction and scattering, GSCM obtain the statistical distribution of the Angle of Arrival/Departure (AOA/AOD) and the delay of Multi-Path Components (MPCs) based on the real Euclidean geometry (Zhiyi et al., 2009) or the Probability Density Function (PDF) of the geometry distribution of the scatterers (Intarapanich et al., 2007). Depending on the propagation channel, a variety of distributions of scatterers on one ring (Abdi, 2004) (one hollow-disk for wideband channels (Latinovic et al., 2003)), around the User Equipment (UE), or two-ring (Tang and Mohan, 2003) (two hollow-disks for wideband channels (Shen et al., 2010)), one around the UE and another one around the Base Station (BS) have been considered in the literature. In addition, three major categories of the propagation channels 1) Outdoor (the scatterers around the BS and the UE are located outdoor), 2) Indoor (the scatterers around the BS and the UE are located indoor), and 3) Outdoor-to-Indoor (O2I) (the scatterers around the BS are located outdoor and the scatterers around the UE are
located indoor) have been recognized. Moreover, the multi-path components can be single-bounce, double-bounce, multiple-bounce, or a combination of all. Despite the fact that real life propagation is dominated by single-bounce multi-path components, multiple-bounce components cannot be neglected when MIMO system performance is investigated (Hofstetter et al., 2006). In the existing geometric models appropriate for O2I, (Abdi, 2004; Latinovic et al., 2003), only single-bounce is considered around a UE to ease the modeling, but this assumption reduces the accuracy of the model.

In this paper, we propose geometrical MIMO channel models for a 2-Dimensional (2D) space of an O2I environment. In order to increase the efficiency of GSCM for wideband systems, the distribution of the scatterers around the UE is a hollow-disk, instead of a ring (Latinovic et al., 2003). First, joint single- and double-bounce multi-path propagation components are modeled where the scatterers are located on a hollow-disk around the UE. Then, the effects of the stationary scatterers and a randomly distributed group of scatterers located outdoor are derived analytically. To verify the models with the one's of a real environment, the propagation channel model of the Electrical and Computer Engineering Department, University of Sistan and Baluchestan, is computed by the Radiowave Propagation Simulator (RPS) package. The mathematical and the simulated models are compared in terms of the cross correlation function (CCF) of MIMO links and the achieved capacity. The CCF and the capacity of the proposed GSCM are close to the ones achieved by the simulator. Moreover, we show that multi-path components are described more accurately by considering the outdoor scatterers.

**Problem Statement**

The proposed channel model featured for 4G systems incorporates the following characteristics:

- To achieve high data rate in 4G networks either more bandwidth or spatial diversity techniques should be deployed. Increasing bandwidth at low frequency applications, such as, LTE-700 applications, is almost impossible due to the lack of free spectrum. A practical solution is to take advantage of channel spatial diversity which can be implemented using MIMO systems.

- Although different antenna elements may exist on each side of a MIMO system, in practice, a simple uniform array with the same elements is used for the BS and the UE. To simplify the analytical calculations and without losing any generality, we consider a $2 \times 2$ MIMO in the geometrical channel modeling.

- Radio channels of 4G systems are usually assumed as O2I channels. As path loss fading dominates at far distances from the BS, we consider only the scatterers surrounding the UE. The effects of the scatterers around the BS are ignored.

- The model is presented in two dimensions, but it can be extended to three dimensions easily. When the operating frequency is low, like the one's of LTE-700, there is a space limitation for the antenna array in the user handset which is investigated in the proposed model.

- Indoor scatterers are located on a hollow-disk around the user, and a group of outdoor scatterers exist between the BS and the UE. The outdoor scatterers have a compact or distributed form.

- Joint single- and double-bounce multi-path propagation, once around the UE and once on the outdoor scatterers, is incorporated to increase the accuracy.

With the aforementioned assumption, we implement the following simulation to give an insight regarding to the effect of double bounce signal propagation around the UE. Using RPS, the O2I environment shown in Fig. 1 is simulated. The indoor propagation channel, where the UE exists, is the one-story building of Electrical and Computer Engineering School at the University of Sistan and Baluchestan. The BS is located outdoor and equipped with an antenna array with two isotropic elements mounted at a height of 5m. The UE has the same antenna array as the one's of the BS at a height of 1.5m.

![Figure 1. The simulated O2I environment](image-url)
RPS considers a finite number of rays received by the user antennas' elements. Accordingly, the channel impulse response between the $p^{th}$ element of the BS and the $l^{th}$ element of the user antenna, is obtained as

$$h_{p}(\tau) = \sum_{i=1}^{n} \sqrt{P_i} e^{j\psi_i} \delta(\tau - \tau_i),$$

where $P_i$, $\psi_i$, and $\tau_i$ are received power, phase shift and delay of the $i^{th}$ received ray, respectively.

Having the impulse response, the frequency response $T_{lp}(f)$ is computed as

$$T_{lp}(f) = \sum_{i=1}^{n} \sqrt{P_i} e^{j\psi_i - j2\pi f \tau_i},$$

and the CCF between the two links of MIMO channel is represented by

$$\rho_{lp,mq}(\Delta f) = \frac{T_{lp}(f)T_{mq}^*(f + \Delta f)}{\sqrt{\Omega_1 \Omega_2}},$$

where $\Omega_1 = \sum_{i=1}^{n_1} P_i$, and $\Omega_2 = \sum_{j=1}^{n_2} P_j$ are used for normalization; $n_1$ and $n_2$ are the number of received rays in related paths.

As a measure of comparison, we use the CCF of three different scenarios considered in this simulation: The propagation environment has 1) only 1 reflection; 2) 1 and 2 reflections; 3) unlimited reflections. The CCF magnitude versus the different antenna spacing are shown in Fig. 2 (the detailed parameters values are given in simulation results section). The results show that adding the double bounce reflections to the multi-path propagation channel improves the CCF significantly. In other words, the CCF of 1 and 2 reflections scenario is very close to the one's of unlimited reflections scenario. Hence, the presented geometric model in this paper emphasizes on considering both single- and double-bounce reflections.

Figure 2. The $|CCF|$, $|\rho_{1,2}(\Delta f)|$, of the simulated model with 1 reflection, 1 and 2 reflections, and unlimited reflections ($\Delta f = 0$ and $\frac{\delta_{pq}}{\lambda} = 1$)

Joint Single- And Double-Bounce On Indoor Scatterers

The geometry of joint single- and double-bounce hollow-disk model is shown in Fig. 3, for a $2 \times 2$ MIMO channel. The user receives signals from different directions reflected on a large number of local scatterers. We
assume planning waves bouncing once or, twice. The $i^{th}$, $k^{th}$ and $i^{th}$ scatterers are represented by $S_i$, $S_k$ and $S_i'$, respectively. $R_i$, $R_k$ and $R_i'$ are the distances between the $i^{th}$, $k^{th}$ and $i^{th}$ scatterers from UE, respectively. $D$ is the distance between the BS and the UE. All scatterers are located on the hollow-disk, with the distance from the user within the range $R_1 < R_i < R_2$, $R_1 < R_k < R_2$, and $R_1 < R_i' < R_2$.

Figure 3. Geometrical configuration of joint single- and double-bounce reflections on a hollow-disk around the UE.

The complex low-pass equivalent channel response between $BS_p$ and $U_l$ is given by $h_{lp}(\tau)$.

The power transferred through the $BS_p-U_l$ link is defined as $\Omega_{lp}$, where $\Omega_{lp} = E[|h_{lp}(\tau)|^2]$, under the assumption of the unit total transmission power. The plane waves are emitted from the array element $BS_p$, travel over different length paths, scattered by the local scatterers around the UE, and impinge the array element $U_l$ on different directions. Since we consider the Rayleigh fading channel, no line of sight (LOS) component exists. Mathematical representation of this propagation mechanism, similar to that of (Fuhl et al., 1998), results in the following expression:

$$h_{lp}(\tau) = \lim_{N \to \infty} \frac{1}{\sqrt{N K}} \sum_{i=1}^{N} \sum_{k=1}^{K} g_{ki}(\zeta_i R_i R_k)^{-n/2} e^{j\psi_{ki} - j2\pi\lambda^{-1}(\zeta_{ip} + \zeta_{ki} + \zeta_{ik})} \delta(\tau - \tau_i) + \lim_{N_i \to \infty} \frac{1}{\sqrt{N_i}} \sum_{i=1}^{N} g_{ii'}(\zeta_{i'} R_i')^{-n/2} e^{j\psi_{ii'} - j2\pi\lambda^{-1}(\zeta_{i'p} + \zeta_{i'i})} \delta(\tau - \tau_{i'})$$

(4)
In the above formula, \( N, K \) and \( N' \) are the number of independent scatterers \( S_i, S_k, \) and \( S_j \), around the UE, respectively, \( g_{ki} \) represents the amplitude of the wave scattered by the \( i^{th} \) scatterer and the \( k^{th} \) scatterer toward the UE, and \( g_{ij} \) represents the amplitude of the wave scattered by the \( i^{th} \) scatterer toward the UE. \( \psi_{ki} \) denotes the phase shift introduced by the \( i^{th} \) and the \( k^{th} \) scatterers and \( \psi_{ij} \) denotes the phase shift introduced by the \( i^{th} \) scatterer. \( \zeta_{ip}, \zeta_{ki}, \zeta_{lk}, \zeta_{ij}, \) and \( \zeta_{il} \) are the distances shown in Fig. 4 and are functions of \( \phi_{iU}, \phi_{kU}, \phi_{jU}, R_i, R_k, \) and \( R_j \). \( \{g_{ki}\}_{k,i=1}^{\infty} \) and \( \{g_{ij}\}_{i=1}^{\infty} \) are independent and identically distributed (i.i.d) random variables with \( E[g_{ki}^2] = 1, \) and \( E[g_{ij}^2] = 1. \) Moreover, \( \{\psi_{ki}\}_{k,i=1}^{\infty} \) and \( \{\psi_{ij}\}_{i=1}^{\infty} \) are i.i.d. random variables with uniform distributions over \([0,2\pi)\). One can easily show that \( \Omega_{lp} = \lim_{N \to \infty} (NK)^{-1}. \)

\[
\sum_{i=1}^{\infty} \sum_{k=1}^{\infty} E[g_{ki}^2](\zeta_{ki} R_k)^{-n} + \lim_{N' \to \infty} (N')^{-1} \sum_{i=1}^{\infty} E[g_{ij}^2](\zeta_{ij} R_j)^{-n}.
\]

The Cross-Correlation Function

The time invariant frequency response, \( T(f) \), is the Fourier transform of \( h(\tau) \), i.e., \( T(f) = \mathcal{F}\{h(\tau)\} \). For the wideband communication link, \( BS_p - U_i \), we have (Bello, 1963)

\[
T_{lp}(\tau) = \lim_{N \to \infty} \lim_{K \to \infty} \frac{1}{\sqrt{NK}} \sum_{i=1}^{N} \sum_{k=1}^{K} g_{ki}(\zeta_{ki} R_k)^{-n/2} e^{j2\pi f \tau_i - j2\pi \lambda^{-1}(\zeta_{ip} + \zeta_{ki} + \zeta_{ik}) + j2\pi f \tau_i} + \lim_{N' \to \infty} \frac{1}{\sqrt{N'}} \sum_{i=1}^{N'} g_{ij}(\zeta_{ij} R_j)^{-n/2} e^{j2\pi f \tau_i - j2\pi \lambda^{-1}(\zeta_{ij} + \zeta_{il} + \zeta_{il}) + j2\pi f \tau_i}.
\]

The normalized CCF between the frequency responses of two arbitrary communication links \( T_{lp}(f) \) and \( T_{mq}(f) \) is defined as (Latinovic et al., 2003):

\[
\rho_{lp,mq}(\Delta f) = E[T_{lp}(f) T_{mq}(f + \Delta f)] / \sqrt{\Omega_{lp} \Omega_{mq}}
\]

(6)

where \( \ast \) denotes the complex conjugate operation. The CCF between \( T_{lp}(f) \) and \( T_{mq}(f) \) can be written as

\[
\rho_{lp,mq}(\tau) = \frac{1}{\Omega} \lim_{N \to \infty} \lim_{K \to \infty} \frac{1}{\sqrt{NK}} \sum_{i=1}^{N} \sum_{k=1}^{K} E[g_{ki}^2](\zeta_{ki} R_k)^{-n} e^{-j2\pi \lambda^{-1}(\zeta_{ip} + \zeta_{iq} + \zeta_{ik} - \zeta_{mk}) + j2\pi f \tau_i} + \frac{1}{\Omega} \lim_{N' \to \infty} \frac{1}{\sqrt{N'}} \sum_{i=1}^{N'} E[g_{ij}^2](\zeta_{ij} R_j)^{-n} e^{-j2\pi \lambda^{-1}(\zeta_{ij} + \zeta_{iq} + \zeta_{il} - \zeta_{mj}) + j2\pi f \tau_i}.
\]

(7)

where \( \Omega_{lp} = \Omega_{mq} = \Omega \) is assumed. For large \( N, K \) and \( N' \), we have
\[E[g_{ki}^2](\zeta_{ki}^2R_k)^{-n}/KN = (\zeta_{i}(\phi_i^U, R_i)\zeta_{ki}(R_iR_k)R_k)^{-n}f(\phi_i^U, R_i)f(\phi_k^U, R_k)dR_i dR_k d\phi_i^U d\phi_k^U\]
\[E[g_{li}^2](\zeta_{li}^2R_l)^{-n}/N' = (\zeta_{i}(\phi_i^U, R_i)R_i)^{-n}f(\phi_l^U, R_l)dR_l d\phi_l^U.\]  
(8)

Note that \(f(\phi_i^U, R)\) is the joint PDF of \(\phi_i^U\) and \(R\).

The distances, \(\zeta_{ip}, \zeta_{iq}, \zeta_{ik}, \zeta_{ip'}, \zeta_{iq'}, \zeta_{ii'},\) and \(\zeta_{mi'}\) are derived using the law of cosines in appropriate triangles:

\[\zeta_{ip}^2 = \left(\frac{\delta_{pq}}{2}\right)^2 + \xi_i^2 - 2\frac{\delta_{pq}}{2}\xi_i \cos(\alpha_{pq} - \phi_i^{BS})\]
\[\zeta_{iq}^2 = \left(\frac{\delta_{pq}}{2}\right)^2 + \xi_i^2 + 2\frac{\delta_{pq}}{2}\xi_i \cos(\alpha_{pq} - \phi_i^{BS})\]
\[\zeta_{ik}^2 = \left(\frac{\delta_{lm}}{2}\right)^2 + R_k^2 - 2R_k \frac{\delta_{lm}}{2} \cos(\phi_k^U - \beta_{lm})\]
\[\zeta_{ip'}^2 = \left(\frac{\delta_{pq}}{2}\right)^2 + \xi_i^2 - 2\frac{\delta_{pq}}{2}\xi_i \cos(\alpha_{pq} - \phi_i^{BS})\]
\[\zeta_{iq'}^2 = \left(\frac{\delta_{pq}}{2}\right)^2 + \xi_i^2 + 2\frac{\delta_{pq}}{2}\xi_i \cos(\alpha_{pq} - \phi_i^{BS})\]
\[\zeta_{ii'}^2 = \left(\frac{\delta_{lm}}{2}\right)^2 + R_k^2 - 2R_k \frac{\delta_{lm}}{2} \cos(\phi_i^U - \beta_{lm})\]
\[\zeta_{mi'}^2 = \left(\frac{\delta_{lm}}{2}\right)^2 + R_k^2 + 2R_k \frac{\delta_{lm}}{2} \cos(\phi_i^U - \beta_{lm})\].
(9)

Similarly, \(\phi_i^{BS}\) and \(\phi_i^{OS}\) are computed using the law of sines in the \(OS_iO'\) and the \(OS_iO'\) triangles, respectively:

\[\frac{\sin(\phi_i^U - \phi_i^{BS})}{D} = \frac{\sin \phi_i^{BS}}{R_i} = \frac{\sin \phi_i^U}{\zeta_i^{'}}\]  
(10)
\[\frac{\sin(\phi_i^U - \phi_i^{BS})}{D} = \frac{\sin \phi_i^{BS}}{R_i} = \frac{\sin \phi_i^U}{\zeta_i^{'}}.\]  
(11)

By the assumption of \(D >> R_2 >> R_1 >> \delta_{lm}\), the equations are simplified drastically. We use the approximated relations \(\sqrt{1 + \frac{X}{2}} \approx 1 + \frac{X}{2}\), \(\sin(X) = X\), and \(\cos(X) \approx 1\) when \(X\) is small (Latinovic et al., 2003). The first equation in (10) yields \(\phi_i^{BS} = \frac{R_i}{D} \sin \phi_i^U\), and the first equation in (11) yields \(\phi_i^{BS} = \frac{R_i}{D} \sin \phi_i^U\). The equations in (9) can be written as
\[ \zeta_{ip} = \zeta_i - \frac{\delta_{pq}}{2} \left[ \cos \alpha_{pq} + \frac{R_i}{D} \sin \alpha_{pq} \sin \phi_i^U \right] \]
\[ \zeta_{ip} = \zeta_i + \frac{\delta_{pq}}{2} \left[ \cos \alpha_{pq} + \frac{R_i}{D} \sin \alpha_{pq} \sin \phi_i^U \right] \]
\[ \zeta_{ik} = R_k - \frac{\delta_{im}}{2} \cos(\phi_k^U - \beta_{im}) \]
\[ \zeta_{mk} = R_k + \frac{\delta_{im}}{2} \cos(\phi_k^U - \beta_{im}) \]
\[ \zeta_{i^*p} = \zeta_i - \frac{\delta_{pq}}{2} \left[ \cos \alpha_{pq} + \frac{R_i^*}{D} \sin \alpha_{pq} \sin \phi_i^U \right] \]
\[ \zeta_{i^*p} = \zeta_i + \frac{\delta_{pq}}{2} \left[ \cos \alpha_{pq} + \frac{R_i^*}{D} \sin \alpha_{pq} \sin \phi_i^U \right] \]
\[ \zeta_{i^*i^*} = R_i^* - \frac{\delta_{im}}{2} \cos(\phi_i^U - \beta_{im}) \]
\[ \zeta_{m^*i^*} = R_i^* + \frac{\delta_{im}}{2} \cos(\phi_i^U - \beta_{im}) \]
\[ \zeta_i = D + R_i \cos(\phi_i^U) \]
\[ \zeta_i^* = D + R_i^* \cos(\phi_i^U) \].

Substitution of (8) and (12) into (7) yields the approximate CCF:
\[
\rho_{ip,mq}(\Delta f) = \frac{1}{\Omega_{ip}^2} \int_{R_i}^{R_i^2} \int_{-\pi}^{\pi} \left( A_1 \right)^{-n} e^{i 2 \pi \Delta f c^{-1} (A_1 - A_2) + j 2 \pi \delta_{pq} \lambda^{-1} (\cos \alpha_{pq} + B_1) + j 2 \pi \delta_{im} \lambda^{-1} B_2} f(\phi_i^U, R_i) \\
\left( \int_{R_i}^{R_i^2} \int_{-\pi}^{\pi} \left( D R_i \right)^{-n} e^{i 2 \pi \delta_{pq} \lambda^{-1} (\cos \alpha_{pq} + j 2 \pi \Delta f c^{-1} (R_i + D) + \delta_{im} \lambda^{-1} (C_1 + \delta_{im} \lambda^{-1} C_2 + R_i \Delta f c^{-1} \cos(\phi_i^U)) \right) f(\phi_i^U, R_i) d\phi_i^U dR_i^e, \right)
\]

where
\[ A_1 = (R_k D^{2} R_i^2 + R_k^2 - 2R_i R_k \cos(\phi_k^U - \phi_i^U)) \]
\[ A_2 = R_i \cos(\phi_i^U) \]
\[ B_1 = \frac{R_i}{D} \sin \alpha_{pq} \sin \phi_i^U \]
\[ B_2 = \cos(\phi_k^U - \beta_{im}) \]
\[ C_1 = \frac{R_i}{D} \sin \alpha_{pq} \sin \phi_i^U \]
\[ C_2 = \cos(\phi_i^U - \beta_{im}). \]

We have assumed that \( \phi^U \) and \( R \) are independent random variables, so \( f(\phi^U, R) = f(\phi^U) f(R) \). Further, we use the Von Mises PDF for the AOA, because it fits very well real data (Abdi et al., 2002):
\[ f(\phi_i^U) = e^{k\cos(\phi_i^U - \mu)} \frac{2\pi l_0(k)}{k}, \quad \phi_i^U \in [-\pi, \pi], \tag{15} \]
\[ f(\phi_i^U) = e^{k\cos(\phi_i^U - \mu)} \frac{2\pi l_0(k')}{k'}, \quad \phi_i^U \in [-\pi, \pi], \tag{16} \]

where \( l_0(.) \) is the zero order modified Bessel function, \( \mu \in [-\pi, \pi] \) accounts for the mean direction of AOA seen by the UE, and \( k' \geq 0 \) and \( k \geq 0 \) control the width of AOA. We substitute the PDFs into (13) and calculate the inner integral according to equation 3.937, p. 496 given in (Gradshteyn and Ryzhik, 2007):
\[ \int_{-\pi}^{\pi} e^{x\sin z + y\cos z} \, dz = 2\pi l_0(\sqrt{x^2 + y^2}) \tag{17} \]
Substitution of \( \phi_k^U - \phi_i^U = \Phi \) into (15) yields the following relation for \( f(\phi_k^U) \):
\[ f(\phi_k^U) = f(\phi_i^U + \Phi) = e^{k\cos(\phi_i^U + \Phi - \mu)} \frac{2\pi l_0(k)}{k}, \tag{18} \]

Therefore, the closed-form expression for CCF is derived:
\[ \rho_{l,p,q}(\Delta\rho) = \frac{1}{2\pi \Omega_{l,p}^2} \int_{R_1}^{R_2} \int_{R_1}^{R_2} (A_1)^{-n} l_0(\sqrt{(x_1 R_i + y_1)^2 + (x_2 R_i + y_2)^2}) e^{j2\pi(\rho_{pq}^{-1} \cos(\alpha_{pq}) + \Delta \rho^{-1} A)} f(R_i) f(R_k) dR_i dR_k + \frac{1}{\Omega_{l,p}} \int_{R_1}^{R_2} (D R_i)^{-n} e^{j2\pi\rho_{pq}^{-1} \cos(\alpha_{pq}) + j2\pi\Delta \rho^{-1} (D + R_i^2)} l_0(\sqrt{(x_1 R_i + y_1)^2 + (x_4 R_i + y_4)^2}) f(R_i^2) dR_i, \tag{19} \]

where
\[ A_1 = (R_k D \sqrt{R_i^2 + R^2_k - 2R_i R_k \cos \Phi}) \]
\[ x_1 = j2\pi\rho_{pq}^{-1} \sin(\alpha_{pq}) D \]
\[ y_1 = -j2\pi\rho_{lm}^{-1} \sin(\Phi - \beta_{lm}) + k \sin \mu - k \sin(\Phi - \mu) \]
\[ x_2 = j2\pi\Delta \rho^{-1} \]
\[ y_2 = j2\pi\rho_{lm}^{-1} \cos \Phi + k \cos \mu + k \cos(\Phi - \mu) \]
\[ y_3 = j2\pi\rho_{lm}^{-1} \sin \beta_{lm} + k' \sin \mu \]
\[ x_4 = j2\pi\Delta \rho^{-1} \]
\[ y_4 = j2\pi\rho_{lm}^{-1} \cos \beta_{lm} + k' \cos \mu. \tag{20} \]

**Joint Single- And Double-Bounce On Indoor And Outdoor Scatterers**

In this section, we consider the effects of outdoor compact and distributed scatterers located between the BS and the UE in the following Subsections, respectively.
**Compact Outdoor Scatterers**

The distance $\zeta_{kl}$ can be computed as

$$\zeta_{kl} = \sqrt{L^2 + R_k^2 - 2R_kL\cos(\alpha - \phi_k^U)},$$

where $L$, $R_k$, $\phi_k^U$ and $\alpha$ are the distances and the angles shown in Fig. 4.

The total transferred power is

$$\Omega = \Omega_1 + \Omega_2,$$

where $\Omega_1$ and $\Omega_2$ are computed as follows:

$$\Omega_1 = \lim_{K \to \infty} \frac{1}{\sqrt{K}} \sum_{k=1}^{K} E[g_k^2](\zeta \sqrt{L^2 + R_k^2 - 2R_kL\cos(\alpha - \phi_k^U)}R_k)^{-n},$$

$$\Omega_2 = \lim_{N' \to \infty} \frac{1}{\sqrt{N'}} \sum_{i'=1}^{N'} E[g_i^2](\zeta_i R_i')^{-n}.$$

Figure 4. Geometrical configuration of joint single- and double-bounce reflections on compact outdoor scatterers and indoor scatterers around the UE

Similar to the previous section, the CCF is obtained for the propagation channel of Figure 4 as
\[
\rho_{lp,mq}(|\Delta f|) = \frac{1}{\Omega} \int_{R_1}^{R_2} \left( \zeta \sqrt{L^2 + R_k^2} - 2R_k L \cos(\alpha - \phi_k^U) \right)^{-n} e^{j2\pi \delta_{pq}^{-1}} \cos(\alpha_{pq} - \Phi) \frac{e^{k \cos(\phi_k^U - \mu)}}{2\pi l_0(k)} \right.
\]
\[
e^{j2\pi \delta_{lm}^{-1}} \cos(\phi_k^U - \beta_{lm}) + j2\pi \delta_{fc}^{-1}(\zeta + \sqrt{L^2 + R_k^2} - 2R_k L \cos(\alpha - \phi_k^U) + R_k) \frac{e^{k \cos(\phi_k^U - \mu)}}{2\pi l_0(k)}
\]
\[
f(R_k) d\phi_k^U dR_k + \frac{1}{\Omega} \int_{R_1}^{R_2} \frac{((D_1 + D_2) R_{\nu})^{-n}}{l_0(k')} e^{j2\pi \delta_{fc}^{-1}(D_1 + D_2 + R_{\nu}) + j2\pi \delta_{pq}^{-1}} \cos(\alpha_{pq} + R_{\nu}) +
\]
\[
l_0(\sqrt{(x_1 R_{\nu} + y_3)^2 + (x_4 R_{\nu} + y_4)^2}) f(R_{\nu}) dR_{\nu},
\]
Distributed Outdoor Scatterers

In this Subsection, outdoor scatterers are distributed on a line shown in Fig. 5. The distance \( \zeta_{kl} \) is calculated as

\[
\zeta_{kl} = \sqrt{L^2 + R_k^2 - 2R_kL\cos(\alpha - \phi_k^U)},
\]

where \( L, R_k, \phi_k^U \) and \( \alpha \) are the distances and the angles shown in Figure 5.

The total transferred power is

\[
\Omega = \Omega_1 + \Omega_2,
\]

where \( \Omega_1 \) and \( \Omega_2 \) are computed as follows:
\[ \Omega_1 = \lim_{N \to \infty} \frac{1}{\sqrt{NK}} \sum_{i=1}^{N} \sum_{k=1}^{K} \mathcal{E}[g_{ik}] (\zeta \sqrt{L^2 + R_k^2 - 2R_kL \cos(\alpha - \phi_k^U)R_k})^{-n} \]

\[ \Omega_2 = \lim_{N' \to \infty} \frac{1}{\sqrt{N'}} \sum_{i'=1}^{N'} \mathcal{E}[g_{i'i'}^2] (\zeta_i R_{i'})^{-n}. \]

Similar to the previous sections and according to the configuration shown in Fig. 5, we have

\[ \rho_{lp, mq}(\Delta f) = \frac{1}{\Omega} \int_{-\pi}^{\pi} \frac{1}{\Omega} \int_{-\pi}^{\pi} e^{j2\pi\delta_{pq}\lambda^{-1}\cos(\alpha_{pq} - \Phi)} \]

\[ f(x)f(R_k)dx d\phi_k^U dR_k + \frac{1}{\Omega} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} e^{j2\pi\delta_{pq}\lambda^{-1}\cos(\alpha_{pq})} \]

\[ I_0(\sqrt{(x_1 R_{i'} + y_3)^2 + (x_4 R_{i'} + y_4)^2}) f(R_{i'}) dR_{i'}. \]

Where

\[ \zeta_i = \sqrt{x^2 + D_1^2}, \quad L = \sqrt{x^2 + D_2^2}, \quad \Phi = \tan^{-1}(\frac{x}{D_1}), \quad \alpha = \pi - \tan^{-1}(\frac{x}{D_2}), \]

and \( h, D_1 \) and \( D_2 \) are the distances shown in Fig. 5.

For \( L >> R_k \) and using the equations (23), the closed-form expression for the CCF is derived:

\[ \rho_{lp, mq}(\Delta f) = \frac{\Omega^{-1}}{2\pi l_0(k)} \int_{l_1}^{l_2} \int_{l_1}^{l_2} e^{j2\pi\delta_{pq}\lambda^{-1}\cos(\alpha_{pq})} I_0(\sqrt{(x_1 R_{i'} + y_3)^2 + (x_4 R_{i'} + y_4)^2}) f(R_{i'}) dR_{i'}. \]

Where

\[ S = \sqrt{(x_5 R_k + y_5)^2 + (x_6 R_k + y_6)^2} \]

\[ F = e^{j2\pi\delta_{pq}\lambda^{-1}\cos(\alpha_{pq})} \sin \Phi + \cos \alpha_{pq} \cos \Phi + j2\pi\delta_{pq}\lambda^{-1}(\zeta + L + R_k) \]

\[ x_5 = -j4\pi\delta_{pq}\lambda^{-1}\cos \alpha \]

\[ y_5 = j2\pi\lambda^{-1}\delta_{im} \cos \beta_{im} + k \cos \mu_1 \]

\[ x_6 = -j4\pi\delta_{pq}\lambda^{-1}\sin \alpha \]

\[ y_6 = j2\pi\lambda^{-1}\delta_{im} \sin \beta_{im} + k \sin \mu_1. \]
SIMULATION RESULTS

In this section, the CCF and the capacity of the three configurations, Joint Single- and Double-Bounce on Indoor scatterers, Joint Single- and Double-Bounce on Indoor and Compact Outdoor Scatterers, and Joint Single- and Double-Bounce on Indoor and Distributed Outdoor Scatterers are investigated. First, the parameters of the derived mathematical CCFs are calibrated according to the CCFs obtained by conducting some simulations on RPS. In other words, the values of $k$ and $\mu$ are determined according to the simulated CCF to project the specifications of the propagation channel into the mathematical CCF. Second, the capacity of the proposed channel models are computed for performance evaluation of each model. The expected capacity is computed according to the following formula:

$$C = E\{\log_2 | I_2 + \frac{SNR}{n} HH^H |\},$$  \hspace{1cm} (28)

where $|.|$ denotes the determinant, $I_2$ is the $2 \times 2$ identity matrix, SNR represents the signal-to-noise ratio, $n$ is the number of antenna elements, $H$ is the channel impulse response matrix and $(.)^H$ denotes the complex conjugate transpose operator. Note that the capacity $C$ is a stochastic process, since the AOA is a random variable (Sarris and Nix, 2007). The following parameters are chosen for the models: the angles of antenna array at the BS and the UE $\alpha_{pq} = \beta_{lm} = \pi/2$, $n = 1.8$, $f = 700MHz$ and $\Delta f = 0$.

Assuming $D = 93\lambda$ and the reasonable assumption of $D >> R_2 >> R_1 >> \delta_{lm}$, the value of the inner and the outer diameter of the disk are computed as $R_1 = 2.3\lambda$, $R_2 = 9.3\lambda$, and $0 < \delta_{lm}/\lambda < 0.25$, where $\frac{\delta_{pq}}{\lambda} = 1$.

**Joint Single- and Double-Bounce on Indoor scatterers**

The mathematical CCF is obtained by calculating the integral of (19), numerically, and the simulated CCF is computed based on equation (3). The calibrated mathematical CCF and the simulated one, shown in Figure 6, are very close. The increasing of the distance of the UE antenna elements, $0 < \delta_{lm}/\lambda < 0.25$, reduces the magnitude of the CCF. Therefore, the channel capacity grows up as shown in Fig. 7. Comparing the capacities of the proposed indoor joint single- and double-bounce model and the simulated one shows that the results are close with negligible error.

Figure 6. The $|CCF|$, $|\rho_{1,21}(\Delta f)|$, of joint single- and double-bounce configuration with indoor scatterers ($k = k' = 12, \mu = \mu' = 1.8, \Phi = \pi$)
Joint Single- and Double-Bounce on Indoor and Compact Outdoor Scatterers

To investigate the impact of compact outdoor scatterers on the propagation channel, we consider a few trees located between the BS and the UE in RPS software, according to Fig. 9. The appropriate values of $k, k', \mu$ and $\mu'$ are set and the resulted mathematical CCFs as well as the simulated CCF are shown in Fig. 10. It is observed that the compact outdoor scatterers reduce the CCFs magnitude. The amount of reduction in theory and simulation results are approximately equal. Using the same parameters, we calculate capacities. Fig. 11 shows that increasing capacity is dependent on reducing the CCF. This fact can be observed more clearly in the simulation results.

Figure 7. The channel capacity of joint single- and double-bounce configuration with indoor scatterers, $(SNR = 100dB, k = k' = 12, \mu = \mu' = 1.8, \Phi = \pi)$

Figure 9. 3D demonstration of the propagation environment with the compact scatterers between the BS and the UE
Figure 10. The $|CCF|$, $\left| \rho_{12} (\Delta f) \right|$, of the configuration with compact outdoor scatterers.

Figure 11. The channel capacity of the configuration with compact outdoor scatterers, $SNR = 100dB$.

**Joint Single- and Double-Bounce on Indoor and Distributed Outdoor Scatterers**

To implement the distributed scatterers in RPS software, we locate layers composed of brick, glass and wood on a line between the BS and the UE as shown in Fig. 12. It is observed from Fig. 13 that with the existence of outdoor distributed scatterers, the CCF of both the mathematical and the simulated models decrease. In addition, the mathematical results are consistent with the simulation ones. According to Fig. 14 capacity increases when the correlation reduces. Note that, the possibility of the BS transmitted rays incidence on the hollow-disk changes when more outdoor scatterers exist, so it is required that the values of $k'$ and $\mu'$ are calibrated again. It is concluded that considering the outdoor scatterers reduces the CCF magnitudes, hence multi-path components are described more accurately and the proposed model characteristics are closer to the real channel specification. To give and insight into the relation of the CCFs and the capacities at different SNRs, in Fig. 15, we present the MIMO channel capacity versus SNR varying from 0 to 30dB. It is observed that the simulated and mathematical
capacity are very close in both with and without outdoor scatterers. Besides, the existence of outdoor scatterers improves the capacity of the MIMO channel.

Figure 12. 3D demonstration of propagation environment with the outdoor distributed scatterers between the BS and the UE

Figure 13. The $|CCF|$, $|\rho_{1,21}(\Delta f)|$, of the configuration with outdoor distributed scatterers
CONCLUSIONS

Enhanced geometrical MIMO propagation channel models for O2I broadband (4G) networks were proposed. First, the effects of assuming both single- and double-bounce reflections on a hollow-disk of scatterers around the UE was investigated. Conducting simulation with RPS showed that the correlation computations accuracy improves with this assumption. Second, the channel impulse responses were achieved, according to the propagation law, when the scatterers were located both indoor, on a hollow-disk around the UE, and outdoor, as compact or distributed scatterers between the BS and the UE. Then the CCF of the proposed scenarios were computed mathematically and calibrated with the information of the simulated ones. According to the CCF parameters, $k$, $k'$, $\mu$ and $\mu'$, the capacity of each scenario were computed. Comparing the mathematical and
simulated values of capacities shows that including the outdoor scatterers, either as a compact form or distributed one, improves the capacity due to the correlation reduction in the MIMO channel.

The proposed MIMO channel model is useful for design, test and analysis of wideband communication networks. In addition, the model is very accurate for studying the channel capacity of frequency-selective MIMO channels under realistic propagation conditions.

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